networks and graphs lecture 4

directed graphs

a digraph consists of a set of vertices and arcs. arcs are like edges but with a head and a tail. a digraph you have *in-degree* and *out-degree*, the *in-degree* is the number of arcs connected to the vertex with its head and the *out-degree* is the number of arcs connected to a vertex with its tail.

> *the sum of the in-degree equals the sum of the out-degree equals the number of arcs.*

representing directed graphs

adjacency matrix

> *a digraphs adjacency matrix has the vertices on the colomns and on the rows.*
>
> *the matrix is most of the time not symmetric but can be.*
>
> *a digraph is called **strict** if for all u,v there must only be one or no arc and for u,u there must not exist an arc(not an arc to itself).*
>
> *the sum of the row of that vertex corresponds to the out-degree, likewise the sum of the column of a vertex corresponds to the in-degree.*

incidence matrix

an incidence matrix for a digraph has the vertices on the rows and the arcs on the columns. a tail of an arc corresponds to 1 and the head of an arc corresponds to -1.

this matrix is not used very often.

directed paths and cycles

a **directed walk** works the same as a walk in an undirected graph but can only **follow the direction of the arcs**.

> *a **directed path** is a the same as an directed walk but now only using **distinct(different) vertices**.*

a **directed cycle** is a directed path but having u=v.

> *two vertices are **strongly connected** if there are directed paths from u to v and from v to u.*

> *two vertices are **weakly connected** if the underlying undirected graph is connected.*

vertex V is **reachable** if there exists a directed (u,v)-path.

orientations

an orientation of a simple graph is a digraph where all edges have been assigned a direction. 2^m orientations exists for a graph with m edges.

> *a **strongly connected orientation** exists if and only if the minimum edge cut is more than or equal to 2.*

edge coloring

each edge connected to a single vertex should have different colors. one of the edge coloring problems is to find the minimal number of colors.

edge chromatic number

the edge chromatic number(Kai prime) is the minimum k where for witch the graph is k-edge color-able.

> *the edge chromatic number either equals delta(G) or delta(G) + 1*
>
> *note: delta(G) means the highest degree of the graph*

vertex coloring

each vertex adjacent to each other must have a different color.

a simple graph G is k-vertex color-able if all neighbor vertices have different colors.

chromatic number

the chromatic number (kai) is the minimum k for the graph to be k-vertex color-able.

> *the chromatic number of a graph is equal or less than delta(G) +1.*
>
> > *the chromatic number for planar graphs is equal to or less than 4.*