Networks and Graphs lecture 3

vertex cuts and edge cuts

vertex cuts

if some set of vertices disconnects the graph into multiple components it is a **vertex cut**

edge cuts

if some set of edges disconnects the graph into multiple components it is a **edge cut**

> *kappa(G) is minimum number of vertices to disconnect graph G*
>
> *lambda(G) is minimum number of edges to disconnect graph G*

formula:

> *kappa(G) is smaller or equal to lambda(G) is smaller or equal to min-degree*
>
> $$\kappa(G) \leq \lambda(G) \leq min\delta$$

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k-connectivity, Harary graphs and Menger's theorem

K-connectivity

> *a graph is k-connected if kappa(G) is bigger or equal to k*

constructing Harary graphs

1. if k = even
   - conciser a circle seat all n vertices ion this circle.
   - connect each vertices to k/2 nearest left-hand vertex. and to its k/2 nearest right-hand vertex
2. if k = odd and n = even
   - construct Hk-1,n
   - add edges: [0,n/2], [1,1+n/2], ... , [(n-2)/2, n-1]
3. if k = odd and n = odd

- construct Hk-1,n
- add edges [0, (n-1)/2], [1,1+(n-1)/2], ... ,[(n-1)/2, n-1]

Menger's theorem

Vertex independence:

> *two paths are **Vertex independence** if they don't share vertices except the start and finish*

Edge independence:

> *two paths are **Edge independence** if they don't share any edges*

Menger's theorem:

> *minimum size of a vertex cut nonadjacent vertices to disconnect u and v == maximum size of vertex independence set*
>
> *minimum size of a edge cut disconnecting vertices u and v == maximum size of edge independent set*

complete graph == a graph with all vertices connected with each-other

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Bipartite graphs and Planar graphs

Bipartite graphs

> *A graph G is **bipartite** if V(G) = V1* union *V2 ->*
>
> 1. *V1* intersection *V2 = NULL and*
> 2. *any **edge** must be connecting a vertex of V1 to a vertex of V2 (there are no edges between vertices of the same V)*

determining bipartite graph

1. pick one vertex
2. pick all neighbors of that vertex except those appearing before in the set
3. repeat until one is left
4. merge even and odd graphs as V1 = even and V2 = odd rows

Planar graphs

a Graph is planar when

> *it is a simple and connected graph where if you draw it in 2D plane, that there are no intersecting edges*

a planar graph divides the plane into sections these are either bounded or unbounded. bounded meaning that it is completely surrounded by edges, and an unbounded one where it is not surrounded by edges.

Euler's formula

with n = vertices, m =edges and r = sections **only for planar graphs**

> $n - m + r = 2$
>
> *another two theorem's devised from Euler's formula:*
>
> $m <= 3n - 6$
>
> *atleast one vertex must be <= 5 for it to be **planar***