

networks and graphs lecture 2

Graph isomorphisms

Are 2 graphs the same?

g_1 and g_2 are isomorphic if there is a one-to-one mapping

all vertices of g_1 must also be all vertices in g_2

if there is a edge from one to another vertex in g_1 it must also be in g_2

algorithms to determine isomorphism:

1. naive algorithm

- try all combinations and test if it holds
- this is very slow, $n = 10$, $n! = 3628800$

2. babai and luks

- relatively fast

how to determine two graphs are not isomorphic?

1. if the number of vertices or edges is not the same in the two graphs
2. if the degree sequence is different for the two graphs

then the two graphs not isomorphic

Havel-Hakimi

let the degree sequence $[s, t_1, t_2, \dots, t_s, d_1, d_2, d_n]$ be an ordered sequence, then the sequence is graphic iff the sequence $[t_1-1, t_2-1, \dots, t_s-1, d_1, \dots, d_n]$ is graphic.

remove highest number and decrease all others connected to it by one (the vertices with the highest degrees)

check if it is graphic, if it cannot be determined repeat this process until it is easy to determine

the proof of this lies in doing the opposite of the algorithm above.

there has to be a simple graph corresponding to the degree sequence,

1. if all neighbours of s are T_i just remove s and all of its edges and we are done
2. if the neighbours set is not exactly T_i , then another must be a neighbour of s which degree must be the same as that of the original vertex. then just swap the labels
3. if the degree is less than the original and s is connected to D_i and T_i is connected with W and W is not connected to D_i then you remove $[s, D_i]$ and

$[Ti, w]$ and add $[s, Ti]$ and $[w, Di]$ (the degree sequence has not changed)

Paths, Cycles and Connectivity

paths and cycles

a (v_0, v_k) - walk sequence is a sequence of vertices and edges that starts in v_0 and ends in v_k with all edges only between the sequence vertices.

a (u, v) - path is a (u, v) -walk sequence of vertices and edges that do not repeat vertices.

a cycle in a simple graph is a (u, u) - path of at least 3 different edges beginning and ending in the same vertex

a simple graph is acyclic if it does not contain a cycle

Connectivity

- two vertices are connected if there is a path between them
- a graph is connected if all vertices are connected

connectivity of vertices is equivalence, = reflexive, symmetric and transitive.

a Component of a graph is the largest pieces of a graph, a graph only has multiple components if the graph is not connected.