Networks and Graphs lecture 11

constructing scale-free networks

in the scale-free networks we mimic the creation of real world networks where new nodes get attached to exiting ones. constructing the scale-free network using a growth process with preferential attachment.(nodes with high degrees will ride in degree and nodes with low degrees will stay low in degree).

BA-Graphs

> *constructing BA-Graphs:*
>
> *let $G \in ER(n_o, p)$ and $V = V(G)$, let $n \gg n_o$ while $|V| < n$ do:*
>
> - *$V \leftarrow V \cup v$ (add new vertex to graph)*
> - *add edges <v,u> for $m \geq n_o$(add edges to the new vertex).*
>
> *each u is chosen with a probability proportional to its degree:*
>
> $$\mathbb{P}[select\ u] = \frac{\delta(u)}{\sum_{w \in V - \{v\}} \delta(w)}$$
>
> *the resulting graph is a $BA(n,n_o;m)$.*

the expected degree distribution of a $BA(n,n_o;m)$-graph is:

$$\mathbb{P}[\delta(u) = k] \approx \frac{2m^2}{k^3} \propto \frac{1}{k^3}$$

> *creating BA-graphs with a specific α:*
>
> *start with a set V of n_o vertices and no edges. while $|V| < n$ do:*
>
> - *$V \leftarrow V \cup v$ (add new vertex).*
> - *add edges <v,u> for $m \geq n_o$. each edge is considered with a probability proportional to $\delta(u)$.(as before)*

> - *for a constant c $\geq$ 0, add c * m edges between vertices V - [v]. the probability to add an edge <x,y> is proportional to $\delta(x) \cdot \delta(y)$.*
>
> In a generalized $BA(n, n_0, m)$-graph, the expected degree distribution is
>
> $$\mathbb{P}[\delta(u) = k] \propto k^{-(2+\frac{1}{1+2c})}$$
>
> For $c = 0$, $\mathbb{P}[\delta(u) = k] \propto \frac{1}{k^3}$
>
> $$\lim_{c \to \infty} \mathbb{P}[\delta(u) = k] \propto \frac{1}{k^2}$$

BA-Graphs have a shorter average path lengths than ER-Graphs, this is due to the presence of hubs(vertices with a very high degree) in the BA-Graphs.

these hubs will be vulnerable to targeted attacks, a scale-free network will quickly become disconnected as hubs are removed. but if the attacks are random a scale-free network will be very robust.

Graphs in $ER(n, \frac{2m}{n-1})$ have the same expected average degree. But they tend to have a much lower clustering coefficient, namely $\frac{2m}{n-1}$.

BA-graphs have a higher clustering coefficient than ER-Graphs, yet these values are still relatively small.