Networks and Graphs lecture 10

complex and random networks

real-world networks can be modeled as a random graph with where edges appear with some probability.

ER-Graph

ER(n,p) is a graph with *n* vertices with the probability *p* of edges to be included in the graph.

> *on average the degree of a vertex is* $p * (n - 1)$.
>
> *on average the clustering coefficient of any vertex in a ER-graph is* ***p***.
>
> *the probability that a vertex is connected to precisely a subset of k vertices is:* $p^k * (1 - p)^{n-1-k}$.
>
> $$\mathbb{P}[\delta(u) = k] = \binom{n-1}{k} \cdot p^k \cdot (1-p)^{n-1-k}$$
>
> For large $G \in ER(n,p)$ the expected average shortest path length $\bar{d}(G)$ tends to
>
> $$\frac{\ln(n) - \gamma}{\ln(\bar{\delta})} + 0.5$$
>
> Where ln is the natural logarithm and γ the Euler's constant (0.577...)

ER-graphs have a small average shortest path length , but don't have a high clustering coefficient.

WS-Graphs

made for constructing a graph with a high clustering coefficient and a small average shortest path length.

> *constructing WS-graphs:*
>
> - *Choose* $n \gg k \gg ln(n) \gg 1$ *and* ***k has to be even***.

> - *position all vertices in a ring and connect all vertices to k/2 right-hand and left-hand neighbors (just like a $H_{n,k}$).*
> - *consider each edge once and with a probability p replace it with an edge randomly chosen and not already connected to the source.*
> - *the resulting graph is a WS(n,k,p)*

For all graphs in $WS(n,k,0)$, the average shortest path length $\bar{d}(u)$ from vertex u to any other vertex is roughly $\frac{n}{2k}$

but when the probability p increases the average shortest path length rapidly decreases while the clustering coefficient also decreases but much slower and thus will the clustering coefficient stay high.

Scale-free networks

$\mathbb{P}[\delta(u) = k]$ decreases exponentially when k increases.

this means that very few nodes have a very high degree. the degree distribution of a scale-free network follows the power-law:

$$\mathbb{P}[\delta(u) = k] \propto k^{-\alpha} \text{(usually } 2 < \alpha < 3\text{)}$$

these networks are called scale-free because when you scale(zoom in of out) you are able to see the same graph.