Resit Assignment 1
Networks and Graphs

May 11, 2018

1 Computation (35%)

Consider a graph G which has $|V(G)| = 10$ and $|E(G)| = 18$. G is *connected*, *Eulerian*, *Hamiltonian* and *bipartite*.

(a) Describe how being "Eulerian" affects the degree of each vertex of the graph.

(b) Create a possible degree sequence for this graph.

(c) What is the minimum number of edges G can have in order to maintain a Hamiltonian Cycle? Motivate your answer.

(d) How many edges would you add to G in order to make it a Complete Graph of 10 vertices (K_{10})?

(e) Draw 2 non-isomorphic representations of the degree sequence found in (b).

2 Induction (20%)

A cyclic graph is a connected graph of 'n' vertices and 'n' edges where every vertex has a degree 2. Cyclic graphs have exactly 1 cycle (2 regions). Triangle, Square, Pentagon, Hexagon, etc are classic examples of cyclic graphs.

Prove, using induction, that a cyclic graph C_n consisting of n nodes and n edges has $\frac{n(n-1)}{2}$ pairs of distinct endpoints (source and destination). *(Note: For this problem, you may assume that a connection from a vertex 'a' to another vertex 'b' is the same as a connection from vertex 'b' to 'a')*

3 Proof (20%)

Let $G = (V, E)$ be a simple graph and let $U \subseteq V$. We define $G_{\oplus}U\{w\}$ to be the graph obtained from G by adding a new vertex w, which is then joined (by creating an edge) to every vertex in U. In other words, $G_{\oplus}U\{w\} = (V \cup \{w\}, E \cup \{\{u, w\} : u \in U\})$.

Prove that if $G = (V, E)$ is a k-connected simple graph and $U \subseteq V$ has size k, then the graph $G_{\oplus}U\{w\}$ is k-connected as well.

4 Graph Theory in Practice (25%)

Amsterdam's data centers are in trouble. A malicious virus, by the name of 'Bace-Fook' has broken into the data centers. You, as a Graph Theory expert, are deployed by the king to visit all the centers and to handle this problem. There are 5 data centers (A, B, C, D, E) which are connected via tunnels. You need some oxygen to travel through the tunnels. The tunnel connections and their oxygen consumption are as follows:

- *A and B are connected via a tunnel which consumes 9 units of oxygen.*
- *A and E are connected via a tunnel which consumes 7 units of oxygen.*
- *B and C are connected via a tunnel which consumes 7 units of oxygen.*
- *B and E are connected via a tunnel which consumes 8 units of oxygen.*
- *B and D are connected via a tunnel which consumes 9 units of oxygen.*
- *C and D are connected via a tunnel which consumes 10 units of oxygen.*
- *D and E are connected via a tunnel which consumes 11 units of oxygen.*

The tunnels are bidirectional. Your mission is to infiltrate all the data centers and inspect all the tunnels to make sure everything is ok. You can only travel between data centers through their tunnels.

(a) Convert the above scenario to a graph. Clearly mention what each node, edge and edge weight represent.

(b) Describe which problem (mentioned in the lecture) does this situation resemble and why?

(c) Given that you have to start and end at A, use your solution of (b) to construct a route which requires minimum oxygen for your mission.