Homework 4
Networks and Graphs

Deadline: May 18, 18:00

Please submit your answers using Canvas
Attach ONE .pdf file with your names, VUnet IDs and homework group names and of course answers!

1 Erdos-Renyi graphs (25%)

Consider an Erdos-Renyi graph in $ER(n, p)$, for any $n \geq 3$ and $p \in [0,1]$. Argue the following facts.

(a) There are in total $\binom{n}{3}$ possible triangles in the graph.

(b) The expected number of edges incident to a vertex u is $p \cdot (n - 1)$.

(c) The expected number of triangles incident to a vertex u is $\frac{p \cdot (n-1) \cdot (p \cdot (n-1) - 1)}{2} \cdot p$

(d) On average the probability that there are no triangles incident to a vertex u is

$$(1 - p)^{\frac{p \cdot (n - 1) \cdot (p \cdot (n - 1) - 1)}{2}}$$

2 Triples (25%)

Definition 1. Binary tree: A tree in which every node has either 0 or 2 children is a binary tree.
Definition 2. Internal node: In a tree, a node is an internal node if it is neither a root nor a leaf node.
Prove (by **induction**) that every binary tree with k internal nodes has $3k+1$ triples.
In other words $n_\Lambda(T) = 3k + 1$ for a binary tree T with k internal nodes.

3 Watts-Strogatz graphs (15%)

Suppose we construct a graph in $WS(n, k, p)$, starting from the n vertices in a ring, where each vertex is connected to its first $k/2$ right-hand and left-hand neighbors.
What is the probability that none of the edges in this original graph is redirected during the construction of the ultimate graph?

4 Barabasi-Albert graphs (15%)

Suppose we are constructing a BA-graph, and that up to now a graph has been built of five vertices v, w, x, y, z with degree 4,3,3,2,2 respectively.
Let a new vertex u be added. What is the probability that the first edge created from u is $\langle u, w \rangle$?

5 Random graphs (20%)

Given an $ER(n, p)$ random graph. How many vertices can we expect to have vertex degree k?