

Homework 3

Networks and Graphs

Deadline: May 11, 18:00

Please submit your answers using Canvas

Attach a single pdf file with your names, VUnet IDs and homework group names and of course answers!

The text of your answers should be typed and not handwritten. Any necessary drawings may be drawn by hand.

1 Trees (20%)

Use **induction** to show that if T is a tree with n vertices, then $\chi(T) \leq 2$.

Note: The point is to use *induction* to show it. Another type of solution, even if correct, will not count. Make every step of induction explicit.

2 Binary trees (20%)

Definition (Binary tree). *A binary tree binary is a tree in which there are exactly two descendants for every node which has a degree greater than 1.*

Note: For further reference, consult the book. You may find that resources online define this differently, so make sure to use the definition given here.

Theorem. *Let T be a nonempty, binary tree. The number of leaf nodes of T is $i + 1$, where i the number of internal nodes.*

Prove the theorem. You may use induction, but this is not compulsory.

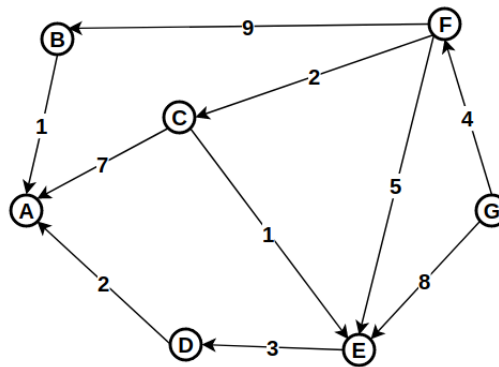
3 Shortest paths, spanning trees (30%)

(a) Use **Dijkstra's** algorithm to compute the sink tree rooted at vertex A in the digraph G_1 . What are the weights of the directed paths to node A for all nodes that can reach it?

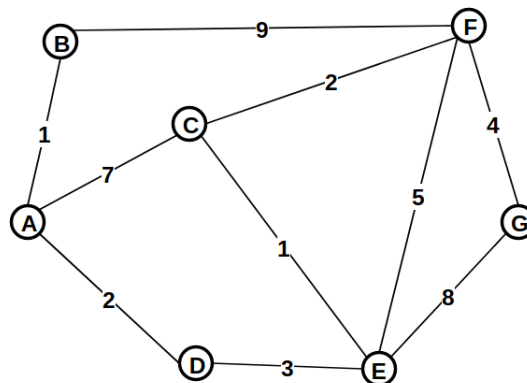
(b) Use **Kruskal's** algorithm to find the minimum weight spanning tree of

the undirected graph G_2 . What is the weight of this spanning tree?
 For both a) and b) describe the intermediate steps of the procedures not just the final solutions!

Digraph G_1



Graph G_2



4 Centrality (30%)

A path P between two distinct vertices in a connected graph G is a diametral path if there is no other path in G whose length is more than the length of P .

- Show that every diametral path in a tree will pass through its central vertices (vertices in the center).
- How can you determine the center of the graph from a diametral path?