Dept. Computer Science VU University Amsterdam	Graphs and Networks 24.03.2010

BE SURE THAT YOUR HANDWRITING IS READABLE

Part I

1 Give the definitions of the following concepts. There is no need to formulate it in math terms, but you do need to be precise!

1a Euler tour *2pt*

1b Vertex-induced subgraph *2pt*

1c Weakly connected directed graph *2pt*

1d Hamilton cycle *2pt*

1e Minimal spanning tree *2pt*

2a Prove that $\forall G, H \subseteq G : |E(H)| \leq |E(G[V(H)])|$. *4pt*

2b Prove that $\exists G, H \subseteq G : |E(H)| > |E(\overline{G[V(H)]})|$. *4pt*

3a Let G be a simple, disconnected graph. Show that the complement $\overline{G}$ of G is connected. *8*

3b Show by example that if G is a simple, connected graph, that $\overline{G}$ can also be connected. *4*

4a Check if the sequence $(7,6,5,4,3,3,2)$ is graphic. Do the same for $(6,6,5,4,3,3,2)$. *6pt*

4b Show that the sequence $(6,6,5,3,3,3,2)$ is graphic by drawing a corresponding graph. *4pt*

5a In the Bellman-Ford algorithm for computing the shortest path $d(v_i, v_j)$ between two vertices v_i, v_j, we update the current value d^t as follows. Explain what is happening. *5pt*

$$d^{t+1}(v_i, v_j) \leftarrow \min_{v_k \in N(v_i)} \{w(v_i, v_k) + d^t(v_k, v_j)\}$$

5b What is the maximal number of rounds needed in the Bellman-Ford algorithm such that every vertex will have discovered the minimal distance to any other vertex? Explain your answer. *5pt*

Part II

6 Give the definitions of the following concepts. There is no need to formulate it in math terms, but you do need to be precise!

6a Center of a graph *2pt*

6b Balanced graph *2pt*

6c Distance between two vertices *2pt*

6d Diameter of a graph *2pt*

7a Prove that for all trees T, the diameter of T is equal to the length of the longest path. *6pt*

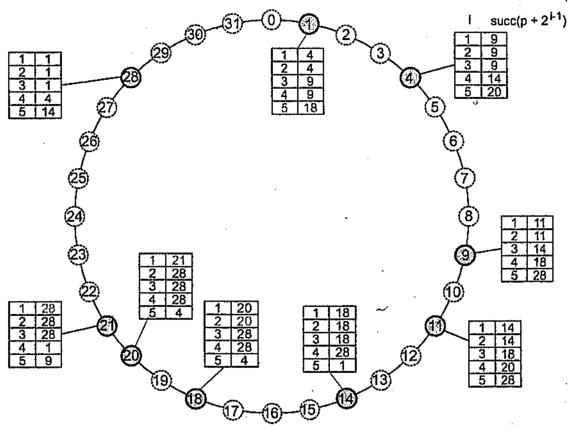
7b Prove that the above statement does not hold in general when T is a simple, connected graph. *4pt*

8a Given an $ER(n,p)$ random graph. How many edges can we expect this graph to have? *4pt*

8b Compute the average degree of a vertex in an $ER(n,p)$ graph. *4pt*

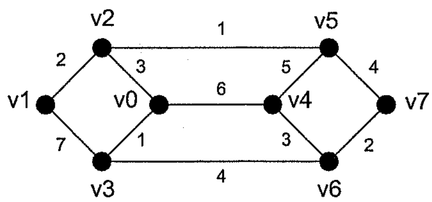
9 Resolve the following key lookups for the shown Chord network.

2@4: look up key 2 at node 4
4@4: look up key 4 at node 4
15@14: look up key 15 at node 14
19@14: look up key 19 at node 14
2@21: look up key 2 at node 21
20@21: look up key 20 at node 21



6pt

10 Compute for each vertex in the following graph *G* its eccentricity, and its clustering coefficient (Hint: clustering coefficients do not depend on weights). Which vertices are in the center of *G*? Which vertices have maximal closeness? *8pt*



11 Consider three people *A*, *B*, and *C* who give relative preference to each other as shown below (where **PREF**$[i,j]$ indicates the relative preference that j gives to i). Compute the ranked prestige of *A*, *B*, and *C*. *10pt*

PREF	A	B	C
A	–	1/5	0
B	1/3	–	1
C	2/3	4/5	–

Final grade: *(1) Add, per part, the total points. (2) Let T denote the total points for the midterm exam* $(0 \leq T \leq 50)$*; D1 the total points for part I; D2 the total points for part II. The final number of points E is equal to* $\max\{T, D1\} + D2$.