Sample Exam I
Multivariate Econometrics
VU Econometrics and Operations Research
2019 – 2020

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Instructions

(i) All questions should be answered to get full points.

(ii) Each question is worth 25 points.

(iii) Read the instructions in the questions carefully.

(iv) Answer the questions as detailed as possible. Use mathematical expressions when necessary. You can use words when you cannot provide a formal mathematical answer to the questions.

(v) If a question is not clear to you, make your own assumptions to clarify the meaning of the question and then answer the question based on your assumptions.

(vi) See the back of this page for some standard results that you may make use of while answering the questions.

Some standard results

Suppose that the scalar process $\{x_t\}$ follows the following data generating process:

$$x_t = x_{t-1} + u_t,$$

where $x_0 = 0$ and u_t has the following properties:

(a) $u_t = \psi(L)\epsilon_t = \sum_{j=0}^{\infty} \psi_j \epsilon_{t-j}$ where $\sum_{j=0}^{\infty} j \cdot |\psi_j| < \infty$ and $\{\epsilon_t\}$ is an $i.i.d$ sequence with mean zero and variance σ_ϵ^2, and finite fourth moment;

(b) σ^2 denotes the long run variance of $\{u_t\}$ and σ_u^2 denotes the contemporaneous variance of $\{u_t\}$.

Let $W(r)$ be a standard Brownian motion process associated with u_t. Then the following results hold:

(1) $T^{-1/2} \sum_{t=1}^{T} u_t \xrightarrow{d} \sigma^2 W(1);$

(2) $T^{-1} \sum_{t=1}^{T} x_{t-1} u_t \xrightarrow{d} \frac{1}{2}\sigma^2 \left[W(1)^2 - \frac{\sigma_u^2}{\sigma^2} \right];$

(3) $T^{-3/2} \sum_{t=1}^{T} t u_{t-j} \xrightarrow{d} \sigma \left\{ W(1) - \int_0^1 W(r) dr \right\}$ for $j = 0, 1, \ldots;$

(4) $T^{-3/2} \sum_{t=1}^{T} x_{t-1} \xrightarrow{d} \sigma \int_0^1 W(r) dr;$

(5) $T^{-2} \sum_{t=1}^{T} x_{t-1}^2 \xrightarrow{d} \sigma^2 \int_0^1 W(r)^2 dr;$

(6) $T^{-5/2} \sum_{t=1}^{T} t x_{t-1} \xrightarrow{d} \sigma \int_0^1 r W(r) dr;$

(7) $T^{-3} \sum_{t=1}^{T} t x_{t-1}^2 \xrightarrow{d} \sigma^2 \int_0^1 r W(r)^2 dr;$

(8) $T^{-(v+1)} \sum_{t=1}^{T} t^v \rightarrow 1/(v+1)$ for $v = 0, 1, \ldots$

Question 1: Conceptual Questions (25 points out of 100 points)

Below you will find 3 statements. All these are related to the concepts/techqniques that have been discussed during the lectures. Some of these statements are correct, some are wrong, some need further clarification. You need to provide a brief, to the point answer that would contain **(i)** short explanations of the concepts mentioned in the statement, **(ii)** your judgement about the statement about whether it is correct/wrong/unclear/incomplete, **(iii)** a correction of the statement. A formal answer using mathematics is possible, sometimes very useful but not always necessary.

(a) (5 points) Mixing and stationarity are important properties that a random sequence might have. About these properties the following is always true: If a process is mixing then it is stationary.

(b) (10 points) Spurious regression is a phenomena we encounter when we regress a random walk on a time trend. The resulting estimator from this regression converges to a random variable and the $t-$statistic of this regression is consistent.

(c) (10 points) Cross sectional dependence is encountered in panel data models. Suppose having the panel data variables $y_{i,t}$ and $x_{i,t}$ for $i = 1, \ldots, N$ and $t = 1, \ldots, T$ with the data generating process

$$y_{i,t} = \beta x_{i,t} + u_{i,t}.$$

The presence of cross sectional dependence implies correlation along time, such that

$$\text{Cov}(u_{i,t} u_{i,s}) \neq 0, \quad \text{for } t \neq s.$$

One way to eliminate the problems that might be induced by cross sectional dependence is to consider the regression of $y_{i,t}$ on $x_{i,t}$ and on the lags of $y_{i,t}$.

Question 2: Modeling and stationarity (25 points out of 100 points)

Suppose that we have the following bivariate data generating process for $\mathbf{w}_t = (y_t, x_t)'$.

$$\mathbf{w}_t = \mathbf{\Gamma}_1 \mathbf{w}_{t-1} + \mathbf{\Gamma}_2 \mathbf{w}_{t-2} + \boldsymbol{\varepsilon}_t,$$

where

$$\mathbf{\Gamma}_1 = \begin{pmatrix} \gamma_{1yy} & \gamma_{1yx} \\ \gamma_{1xy} & \gamma_{1xx} \end{pmatrix}, \quad \mathbf{\Gamma}_2 = \begin{pmatrix} \gamma_{2yy} & \gamma_{2yx} \\ \gamma_{2xy} & \gamma_{2xx} \end{pmatrix}$$

and

$$\boldsymbol{\varepsilon}_t = \begin{pmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{pmatrix} \sim IN \left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{pmatrix} \right],$$

for $t = 1, \ldots, T$. This model can be written in a vector error correction model (VECM) form as

$$\Delta \mathbf{w}_t = \mathbf{A} \mathbf{w}_{t-1} + \mathbf{B} \Delta \mathbf{w}_{t-1} + \boldsymbol{\varepsilon}_t.$$

Answer the following questions.

(a) (10 points) Starting from the VAR(2) model derive the VECM and write $\mathbf{A}$ and $\mathbf{B}$ in terms of the parameters of the VAR(2).

(b) (10 points) Now consider the vector error correction model. Suppose that $\mathbf{A}$ can be written as

$$\mathbf{A} = \boldsymbol{\alpha} \boldsymbol{\beta}',$$

where $\boldsymbol{\alpha}$ is 2×1 and $\boldsymbol{\beta}$ is 2×1 and has the form

$$\boldsymbol{\alpha} = \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix}, \quad \boldsymbol{\beta} = \begin{pmatrix} 1 \\ -\beta_1 \end{pmatrix}.$$

Furthermore, let

$$\mathbf{B} = \begin{pmatrix} b_{11} & b_{12} \\ 0 & b_{22} \end{pmatrix}.$$

Derive the conditional error correction model of y_t given x_t and the past. How would you test for no-cointegration in this CECM. Under what condition(s) is x_t weakly exogeneous for the parameters of interest $\phi = \{\alpha_1, \beta_1\}$.

(c) (5 points) Now let $\gamma_{1yx} = 0$ and $\gamma_{2yx} = 0$ and consider the model for y_t only. Suppose that $\gamma_{1yy} = 0.7$ and $\gamma_{2yy} = -0.1$. Is y_t stationary under these restrictions? Show your calculations.

Question 3: Asymptotic Derivations (25 points out of 100 points)

Suppose that we have the following data generating process for $\{y_t\}$ and $\{x_t\}$:

$$y_t = \mu + y_{t-1} + \varepsilon_{1,t}$$
$$x_t = \delta t + \varepsilon_{2,t},$$

for $t = 1, \ldots, T$. We assume that $\varepsilon_{1,t}$ and $\varepsilon_{2,t}$ satisfy the same assumptions as the error process u_t that are indicated in **(a)** in Page 2. Additionally, we assume:

(i) $\varepsilon_{1,t}$ and $\varepsilon_{2,t}$ are uncorrelated with each other;

(ii) Long run variances of $\varepsilon_{1,t}$ and $\varepsilon_{2,t}$ are σ_1^2 and σ_2^2, respectively;

(iii) $y_0 = 0$ and $x_0 = 0$;

(iv) μ and δ are non-zero constants.

We consider the least squares estimation of the following model:

$$y_t = \beta x_t + e_t,$$

using a sample of T observation pairs. Answer the questions below.

(a) (10 points) Derive and discuss the orders of probability and limiting distributions of

$$\sum_{t=1}^{T} x_t^2, \quad \sum_{t=1}^{T} x_t y_t.$$

(b) (5 points) Derive the limiting distribution of the OLS estimator $\hat{\beta}$. Interpret your results.

(c) (10 points) The $t-$statistic to test for the significance of β can be written as

$$t_{\beta=0} = \left(\sum_{t=1}^{T} x_t^2 \right)^{1/2} \hat{\beta}\, \hat{\sigma}_e^{-1},$$

where $\hat{\sigma}_e^2 = \frac{1}{T} \sum_{t=1}^{T} (y_t - \hat{\beta} x_t)^2$. Derive the limiting distribution of this $t-$statistic. Interpret your result. (Note: use words if you cannot provide a formal mathematical answer.)

Question 4: Empirical Application (25 points out of 100 points)

(a) An econometrics student from the VU wants to analyze money demand in the Netherlands using annual economic time series for the period 1917 – 2017. The variables she uses are the log of money stock (denoted by m_t), the log of the price index (denoted by p_t), the log of real GDP (denoted by y_t) and the central bank discount rate (denoted by r_t).

(i) (5 points) She runs Dickey-Fuller tests (based on regression models with both a constant and a trend: $x_t = \alpha + \delta t + \rho x_{t-1} + u_t$) for the levels of m_t, p_t, y_t and r_t and also for the first differences of these series. The outcomes are as follows

Variables	$t-$stat levels	$t-$stat first differences	2.5 % $t-$distribution critical values	2.5% Dickey-fuller critical values
m_t	-2.04	-4.30	-1.98	-3.95
p_t	-2.98	-7.23	-1.98	-3.95
y_t	-1.45	-4.00	-1.98	-3.95
r_t	-0.94	-3.97	-1.98	-3.95

Explain why she is doing this analysis. What is being tested? What is the test regression? What are the null hypothesis and the alternative hypothesis? How can we draw conclusions from the results in the table? What are the conclusions?

(ii) (10 points) In her report she reports the following regression result as well:

$$\widehat{\Delta m_t} = \underset{(0.03)}{-0.12}\,(m_{t-1} - \underset{(0.56)}{3.12} y_{t-1} - \underset{(0.03)}{0.18} r_{t-1}) + \underset{(0.34)}{0.68} \Delta m_{t-1} + \underset{(0.20)}{0.10} \Delta p_{t-1},$$

$R^2 = 0.56$, where the numbers in the parentheses are the associated standard errors. Explain why she reports this result. What is she investigating? Interpret the results. Is it correct to estimate this single equation model? Do you have any suggestions for alternative models and estimators that would improve the accuracy of the conclusions?

(b) Another econometrics student from the VU is analyzing a panel data set on yearly housing expenses of 434 families over 5 years. Previous research shows that housing expenses data is dynamic in nature due to the presence of the lagged version of this variables in its model and it is highly suspected that an individual specific effect is present in the error term of the model that represents the heterogeneity among the families. The student realizes these and assumes the following model, where he denotes the variable by $c_{i,t}$:

$$c_{i,t} = \lambda c_{i,t-1} + u_{i,t},$$

where $u_{i,t} = \mu_i + \varepsilon_{i,t}$. Then she regresses $\widetilde{c}_{i,t} = c_{i,t} - T^{-1}\sum_{t=1}^{T} c_{i,t}$ on $\widetilde{c}_{i,t-1} = c_{i,t-1} - \sum_{t=1}^{T} c_{i,t-1}$ and obtains the pooled OLS estimator as

$$\widehat{\lambda}_{FE} = \frac{\sum_{i=1}^{N}\sum_{t=1}^{T} \widetilde{c}_{i,t}\widetilde{c}_{i,t-1}}{\sum_{i=1}^{N}\sum_{t=1}^{T} \widetilde{c}_{i,t-1}^{2}}.$$

She finds that $\widehat{\lambda}_{FE} = 0.8$. She thinks that she can rely on this result to say something reliable about the dimension of the dynamic nature of household consumption.

(i) (5 points) Describe the situation faced by this student. Motivate the use of $\widehat{\lambda}_{FE}$ as an estimator. What is the problem with this estimator? When does it occur? Why does it occur? Explain in detail.

(ii) (5 points) Propose and discuss briefly an alternative estimator that is more proper/optimal to be used in this setup.