Sample Exam II
Multivariate Econometrics
VU Econometrics and Operations Research
2019 – 2020

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Instructions

(i) All questions should be answered to get full points.

(ii) Each question is worth 25 points.

(iii) Read the instructions in the questions carefully.

(iv) Answer the questions as detailed as possible. Use mathematical expressions when necessary. You can use words when you cannot provide a formal mathematical answer to the questions.

(v) If a question is not clear to you, make your own assumptions to clarify the meaning of the question and then answer the question based on your assumptions.

(vi) See the back of this page for some standard results that you may make use of while answering the questions.

Some standard results

Suppose that the scalar process $\{z_t\}$ follows the following data generating process:

$$z_t = z_{t-1} + u_t,$$

where $z_0 = 0$ and u_t has the following properties:

(a) $u_t = \psi(L)\epsilon_t = \sum_{j=0}^{\infty} \psi_j \epsilon_{t-j}$ where $\sum_{j=0}^{\infty} j \cdot |\psi_j| < \infty$ and $\{\epsilon_t\}$ is an $i.i.d$ sequence with mean zero and variance σ_ϵ^2, and finite fourth moment;

(b) σ^2 denotes the long run variance of $\{u_t\}$ and σ_u^2 denotes the contemporaneous variance of $\{u_t\}$.

Note that under these assumptions z_t can be written as a partial sum as

$$z_t = \sum_{s=1}^{t} u_s.$$

Let $W(r)$ be a standard Brownian motion process associated with u_t. Then the following results hold:

(1) $T^{-1/2} \sum_{t=1}^{T} u_t \xrightarrow{d} \sigma W(1)$;

(2) $T^{-1} \sum_{t=1}^{T} u_t^2 \xrightarrow{p} \sigma_u^2$;

(3) $T^{-1} \sum_{t=1}^{T} z_{t-1} u_t \xrightarrow{d} \frac{1}{2}\sigma^2 \left[W(1)^2 - \frac{\sigma_u^2}{\sigma^2} \right]$;

(4) $T^{-3/2} \sum_{t=1}^{T} t u_{t-j} \xrightarrow{d} \sigma \left\{ W(1) - \int_0^1 W(r)dr \right\}$ for $j = 0, 1, \ldots$;

(5) $T^{-3/2} \sum_{t=1}^{T} z_{t-1} \xrightarrow{d} \sigma \int_0^1 W(r)dr$;

(6) $T^{-2} \sum_{t=1}^{T} z_{t-1}^2 \xrightarrow{d} \sigma^2 \int_0^1 W(r)^2 dr$;

(7) $T^{-5/2} \sum_{t=1}^{T} t z_{t-1} \xrightarrow{d} \sigma \int_0^1 r W(r)dr$;

(8) $T^{-3} \sum_{t=1}^{T} t z_{t-1}^2 \xrightarrow{d} \sigma^2 \int_0^1 r W(r)^2 dr$;

(9) $T^{-(v+1)} \sum_{t=1}^{T} t^v \rightarrow 1/(v+1)$ for $v = 0, 1, \ldots$

Question 1: Conceptual Questions (25 points out of 100 points)

Below you will find 3 statements. All these are related to the concepts/techqniques that have been discussed during the lectures. Some of these statements are correct, some are wrong, some need further clarification. You need to provide a brief, to the point answer that would contain **(i) short explanations/definitions of the concepts mentioned in the statement, (ii) your judgement about the statement about whether it is correct/wrong/unclear/incomplete, and an explanation of your judgement (iii) a correction of the statement.** The concepts that you need to explain and define are written in italics. A formal answer using mathematics is possible, sometimes very useful but not always necessary.

(a) (10 points) *Granger causality* and *strong exogeneity* imply *super exogeneity.*

(b) (5 points) Consider the following model for $\{y_t\}$:

$$y_t = \rho y_{t-1} + \epsilon_t,$$

for $t = 1, \ldots, T$. We can *test for a unit root in* y_t, by estimating the model by OLS and testing the hypothesis, $\mathcal{H}_0 : \quad \rho = 1$ against the alternative $\mathcal{H}_1 : \quad |\rho| < 1$. The *test statistic obtained from this regression* will have a $t-$distribution in finite samples regardless of the serial correlation structure of ϵ_t.

(c) (10 points) *Nickell bias* in *dynamic panel data models* is caused by the fixed effects transformation to eliminate *cross-sectional dependence.* A way to eliminate the Nickell bias is to estimate directly by OLS the first differenced model.

Question 2: Modeling and stationarity (25 points out of 100 points)

Suppose that we have the following bivariate error correction model for $\mathbf{w}_t = (y_t, x_t)'$.

$$\Delta\mathbf{w}_t = \mathbf{\Pi}\mathbf{w}_{t-1} + \mathbf{\Gamma}\Delta\mathbf{w}_{t-1} + \boldsymbol{\varepsilon}_t,$$

where $\mathbf{\Pi}$ and $\mathbf{\Gamma}$ are 2×2 matrices. and

$$\boldsymbol{\varepsilon}_t = \begin{pmatrix} \varepsilon_{y,t} \\ \varepsilon_{x,t} \end{pmatrix} \sim IN\left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{pmatrix}\right],$$

for $t = 1, \ldots, T$. This model can be written in a vector moving average form (VMA) form as

$$\mathbf{w}_t = \mathbf{A}(L)\boldsymbol{\varepsilon}_t,$$

where $\mathbf{A}(L) = \sum_{i=0}^{\infty} \mathbf{A}_i L^i$. Answer the following questions.

(a) (10 points) Starting from the VECM model derive the VMA and write $\mathbf{A}_i$ for $i = 1, 2, \ldots$ in terms of the parameters of the VECM.

(b) (5 points) Consider $\mathbf{\Pi}$ and answer the following questions.

- **(i)** Let rank$[\mathbf{\Pi}] = 0$. What does this imply for the elements of $\mathbf{w}_t$?
- **(ii)** Let rank$[\mathbf{\Pi}] = 1$. What does this imply for the elements of $\mathbf{w}_t$?
- **(iii)** Let rank$[\mathbf{\Pi}] = 2$. What does this imply for the elements of $\mathbf{w}_t$?

(c) (10 points) Let the rank of $\mathbf{\Pi}$ be equal 1 and consider the decomposition

$$\mathbf{\Pi} = \boldsymbol{\alpha}\boldsymbol{\beta}'$$

where $\boldsymbol{\alpha}$ is 2×1 and $\boldsymbol{\beta}$ is 2×1 and has the form

$$\boldsymbol{\alpha} = \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix}, \quad \boldsymbol{\beta} = \begin{pmatrix} 1 \\ -\beta_1 \end{pmatrix}.$$

Furthermore, let

$$\mathbf{\Gamma} = \begin{pmatrix} \gamma_{yy} & \gamma_{yx} \\ \gamma_{xy} & \gamma_{xx} \end{pmatrix},$$

(i) Derive the conditional error correction model (CECM) of y_t given x_t and the past.

(ii) How would you test for no-cointegration in this CECM.

(iii) Under what condition(s) is x_t weakly exogeneous for the parameters of interest $\phi = \{\alpha_1, \beta_1\}$.

(iv) Propose a practical method to test for the weak exogeneity of x_t for the parameters of interest.

Question 3: Asymptotic Derivations (25 points out of 100 points)

(a) (10 points) Suppose that we have the following data generating process for $\{y_t\}$:

$$y_t = \delta + y_{t-1} + u_t$$

for $t = 1, \ldots, T$. We assume:

- $u_t = \psi(L)\epsilon_t = \sum_{j=0}^{\infty} \psi_j \epsilon_{t-j}$ where $\sum_{j=0}^{\infty} j \cdot |\psi_j| < \infty$ and $\{\epsilon_t\}$ is an $i.i.d$ sequence with mean zero and variance σ_ϵ^2, and finite fourth moment;
- σ^2 denotes the long run variance of $\{u_t\}$ and σ_u^2 denotes the contemporaneous variance of $\{u_t\}$;
- $y_0 = 0$;
- δ is a non-zero constant.

We consider the following regression model:

$$y_t = \mu t + e_t,$$

using a sample of T observation pairs. Consider the least squares estimator

$$\widehat{\mu} = \frac{\sum_{t=1}^{T} y_t t}{\sum_{t=1}^{T} t^2}.$$

Derive and discuss the orders of probability and limiting distributions of the numerator and the denominator of $\widehat{\mu}$. Derive the limiting distribution of the OLS estimator $\hat{\mu}$. Interpret your results. Does it make sense to use $\widehat{\mu}$ as an estimator for δ? Explain.

(b) (15 points) Suppose that we have the following data generating process for $\{y_t\}$:

$$y_t = \rho y_{t-1} + u_t$$

for $t = 1, \ldots, T$. We assume:

- $u_t = \psi(L)\epsilon_t = \sum_{j=0}^{\infty} \psi_j \epsilon_{t-j}$ where $\sum_{j=0}^{\infty} j \cdot |\psi_j| < \infty$ and $\{\epsilon_t\}$ is an $i.i.d$ sequence with mean zero and variance σ_ϵ^2, and finite fourth moment;
- σ^2 denotes the long run variance of $\{u_t\}$ and σ_u^2 denotes the contemporaneous variance of $\{u_t\}$;
- $y_0 = 0$;
- $\rho = 1$.

The t-statistic for

$$\mathcal{H}_0 : \rho = 1,$$
$$\mathcal{H}_A : |\rho| < 1,$$

can be written as

$$t_{\rho=1} = \frac{\widehat{\rho} - 1}{\sqrt{\widehat{\sigma}_u^2 / \sum_{t=1}^{T} y_{t-1}^2}},$$

where $\widehat{\rho}$ is the OLS type estimator that has the form

$$\widehat{\rho} = \frac{\sum_{t=1}^{T} y_{t-1} y_t}{\sum_{t=1}^{T} y_{t-1}^2},$$

where $\widehat{\sigma}_u^2$ is the residual variance estimator that satisfies

$$\widehat{\sigma}_u^2 \xrightarrow{p} \sigma_u^2.$$

Answer the following questions.

(i) Find the order of probability and the limiting distribution of $t_{\rho=1}$ under the null hypothesis.

(ii) Find the order of probability of $t_{\rho=1}$ under the alternative hypothesis.

(iii) Is this t-statistic consistent?

Question 4: Empirical Application (25 points out of 100 points)

(a) (10 points) An econometrics student from the VU wants to analyze the evolution of CO2 level and its relation with economic activiy using long annual time series for the period 1918 - 2018. The retained variables are the log of CO2 emissions ($\ln c_t$), the log of domestically produced goods in the Netherlands ($\ln x_t$). He first tests for a unit root in the log of domestically produced goods and concludes that it has a unit root.

Then, his analysis continues by fitting some ARMA models to the series. He obtains the following results

$$(1 - \underset{(0.5)}{1.5}\ L - \underset{(0.20)}{0.5}\ L^2)\ \ln c_t = (1 + \underset{(0.02)}{0.80}\ L)\ \widehat{\varepsilon}_t.$$

Standard errors are given in parentheses.

Third step in his analysis is to estimate a potential long run relation between the logarithm of CO2 emissions and log of domestically produced goods by OLS. This yields the following result

$$\widehat{ln\ x_t} = \underset{(0.30)}{-4.32} \quad + \quad \underset{(0.21)}{0.63}\ ln\ c_t.$$

An ADF test on the residuals yields the test statistic -2.46. He concludes that there exists a long run relation between the CO2 emissions and domestically produced goods.

You are asked to interpret the results reported above and comment on the appropriateness of his analysis. In particular,

(i) Calculate the roots of the MA and of he AR polynomials. Given these, comment on the stability and stationarity and invertibility of the series.

(ii) Is there evidence in favor of the existence of a long run relation between CO2 emissions and domestically produced goods? Explain.

(iii) Can you use this static least squares regression to test the null hypothesis of unit CO2 elasticity of domestically produced goods? If not, why not? If not, what would you propose?

(b) (15 points) Another econometrics student from the VU is analyzing a panel data set of real house prices and real income for 134 countries over 145 quarters. Let $p_{i,t}$ denote the real house price in country i at time period t and $y_{i,t}$ denote the real income in country i at time period t. The econometrician considers the model

$$p_{i,t} = \beta_i y_{i,t} + u_{i,t},$$

where $\beta_i = \beta + \eta_i$ with $\eta_i \sim i.i.d(0, \sigma_\eta^2)$.

He suspects that there might be correlation between the error terms of the models for different countries, such that

$$Cov(u_{i,t}u_{j,t}) \neq 0, \text{for } i \neq j.$$

He believes that this correlation is due the presence of an unobserved common shock that affects the house prices of all countries. He assumes

$$u_{i,t} = \lambda_i f_t + e_{i,t}, \tag{1}$$

where $e_{i,t}$ is independently and identically distributed across i and t. Furthermore he suspects that the same factor affects the real incomes of all the countries so he assumes

$$y_{i,t} = \gamma_i f_t + \epsilon_{i,t}, \tag{2}$$

where $\epsilon_{i,t}$ is independently and identically distributed across i and t.

He starts looking for an advice on what would be the effects of ignoring f_t and on how to proceed in this situation.

(i) Inform the econometrician about the consequences of the presence of the unobserved f_t in (1)–(2).

(ii) Pesaran (2006) proposes a method to estimate β_i and β in this set up. Discuss the method he proposes. Discuss the important assumptions of this method.