Test 1, Multivariable Calculus, March 29, 2023

Name _____

Student number _____

The use of books, internet enabled devices, including smart phones and tablets, is not permitted.

Please write all work and answers with PEN.
Points: 1. (5 pts), 2. (10 pts), 3. (10 pts), 4. (15 pts), 5. (10 pts), 6. (15 pts), 7. (15 pts), 8. (10 pts), 9. (10 pts).

ESSAY. Write your work and answer on a separate sheet.

1) Find the volume of a parallelepiped spanned by vectors from the origin to the three points $(1, 1, -3)$, $(-1, 3, -1)$, and $(3, 5, 7)$.

2) Find the perimeter of the astroid curve $x = 2 \cos^3 \theta, \ y = 2 \sin^3 \theta$.

3) Find the Taylor polynomial of degree 2 for the function $(x + y)^2 \ln (x + y)$ about the point $(0, 1)$.

4) Do the equations $x^2 + y^2 s + y t^2 = 13$ and $y^2 + x^2 s + x t^2 = 9$ define s and t as functions of x and y near the point $(x, y, s, t) = (1, 2, 1, -2)$? If so, find $\dfrac{\partial s}{\partial y}$ at that point.

5) Evaluate $\displaystyle\int\!\!\int_R y^2 \ dA$, where R is the region bounded by $y = 2x, \ y = 5x, \ $ and $x = 2$.

6) Find the maximum and minimum values of the function $f(x,y) = x^2 + y^2$ on the ellipse $5x^2 + 6xy + 5y^2 - 8 = 0$.

7) Find the distance between the lines $\dfrac{x}{2} = \dfrac{y-2}{2} = \dfrac{z-2}{-1}$ and $\dfrac{x-3}{2} = \dfrac{y-3}{-1} = \dfrac{z-1}{2}$.

8) Use polar coordinates to evaluate $\displaystyle\int_0^1 \int_0^{\sqrt{1-x^2}} \left(x^2 + y^2\right)^{5/2} \ dy \ dx$.

9) Evaluate the integral $\displaystyle\int\!\!\int\!\!\int_R 30xy \ \ dV$, where R is the region defined by the inequalities

$0 \le y \le 1, \ 0 \le z \le y, \ 0 \le x \le z$.