Midterm 1, Multivariable Calculus, March 24, 2021

Name _____

Student number _____

The use internet enabled devices, including smart phones and tablets, is not permitted.

Please write all work and answers with PEN.
Points: 1. (10 pts), 2. (10 pts), 3. (10 pts), 4. (10 pts), 5. (10 pts), 6. (10 pts), 7. (10 pts), 8. (15 pts), 9. (15 pts).

ESSAY. Write your work and answer on a separate sheet.

1) Find the arc length of the space curve given by the vector equation $\mathbf{r} = t\,\mathbf{i} + 2\ln(t)\,\mathbf{j} + (1 - \frac{2}{t})\,\mathbf{k}$, $1 \le t \le 2$.

2) Find the two unit vectors orthogonal to both $\mathbf{a} = 3\mathbf{j} + 2\mathbf{k}$ and $\mathbf{b} = -\mathbf{i} - 2\mathbf{k}$.

3) Find the Taylor polynomial of degree 2 for the function $f(x, y) = \frac{1}{x}e^y$ expanded around (x,y)=(1,0).

4) Find the distance between the lines $\frac{x}{2} = \frac{y-2}{2} = \frac{z-2}{-1}$ and $\frac{x-3}{2} = \frac{y-3}{-1} = \frac{z-1}{2}$.

5) Let $z = f(x,y) = x^y + y$ where $x = u^2 + v^2 - 4$ and $y = uv - 1$. Find the value of $\frac{\partial z}{\partial u}$ at $(u, v) = (2, -1)$.

   Hint: $x^y = e^{y\ln(x)}$

6) Use Lagrange multipliers to find the maximum and minimum values of the function $f(x, y) = xy^2$ on the circle $x^2 + y^2 = 3$.

7) Evaluate $\displaystyle\int\int_R y^2\, dA$, where R is the region bounded by $y = 2x$, $y = 5x$, and $x = 2$.

8) Evaluate the integral $\displaystyle\int_0^{3/2}\int_{x\sqrt{3}}^{\sqrt{9-x^2}} 2xy\, dy\, dx$ using polar coordinates.

9) Evaluate $\displaystyle\int\int\int_E 15\,x^2\, dV$, where E is the region in 3-space described by the inequalities $0 \le x \le 2 - y - z$, $0 \le z \le 2 - y$, and $0 \le y \le 2$.