

Exam Mathematical Economics 3, March 2021

Question 1: Games and Networks (24 points)

Consider a market with one seller, say player 1, who owns a house that is for sale. There are two potential buyers for the house, players 2 and 3. Player 2 has valuation 4 for the house, while player 3 has valuation 8 for the house. The house can only be sold through a real estate agent, player 4. The seller has reservation value zero for the good. Also the valuation of the real estate agent for the good is zero.

This situation can be modeled by a graph game (v, L) on player set $N = \{1, 2, 3, 4\}$, with graph L given by $L = \{\{1, 4\}, \{2, 4\}, \{3, 4\}\}$ and characteristic function v given by

$$v(S) = \begin{cases} 4 & \text{if } S \in \{\{1, 2\}, \{1, 2, 4\}\} \\ 8 & \text{if } S \in \{\{1, 3\}, \{1, 2, 3\}, \{1, 3, 4\}, \{1, 2, 3, 4\}\} \\ 0 & \text{otherwise} \end{cases}$$

- a. (10 points) Give the Myerson restricted game v_L , and compute the Myerson value of this game. Hint: To write down the restricted game v_L it is sufficient to mention which coalitions have a nonzero worth, and what is their worth.
- b. (10 points) Compute the four hierarchical outcomes of this game (so, compute $h^i(v, L), i \in \{1, 2, 3, 4\}$), and verify if they belong to the Core of the Myerson restricted game v_L .

Suppose that the valuation of buyer 2 decreases to an unknown number $0 < v_2 < 4$.

- c. (4 points) Argue that for every value $0 < v_2 < 4$, the Myerson value does not belong to the Core of the restricted game v_L .

Question 2: Games and Networks (8 points)

Let $\Omega \subseteq 2^N$ be the collection of connected coalitions in an undirected graph (N, L) . Show that Ω satisfies union stability. (In other words, show that $S, T \in \Omega$ with $S \cap T = \emptyset$ implies that $S \cup T \in \Omega$.)

Question 3: Search and Matching (16 points)

(a) **Bargaining (Binmore, Rubinstein Wolinsky)** Consider the bargaining game of Binmore, Rubinstein and Wolinsky (BRW).

Let z be the outside option of the firm and b the outside option of the worker and y match output. If an offer is rejected, a breakdown occurs with probability p . Assume that there is enough surplus for trade to take place so that the unique SPE is immediate agreement. Let x_w^* be the highest wage that a firm would accept and x_f^* , the lowest wage that a worker would accept.

- a. (4 points) Explain briefly in words (max 6 lines), why the equilibrium wage offers under non-transferable utility in BRW can be written as,

$$\begin{aligned} y - x_w^* &= pz + (1 - p)(y - x_f^*) \\ x_f^* &= pb + (1 - p)x_w^* \end{aligned}$$

(b) **From discrete to continuous time.** Let b be the value of leisure, suppose the future is discounted at rate: $\frac{1}{1+r}$, let the matching rate be m and denote the value of employment by V_E .

- b. (4 points) Explain that the discrete time Bellman equation (or value function) for the state of unemployment is,

$$V_U = b + \frac{1}{1+r} (mV_E + (1-m)V_U)$$

- c. (8 points) Derive the continuous time Bellman equation.

Question 4: Collusion and Competition Policy (16 points)

Assume that there are two identical firms in the market producing q_1 and q_2 , respectively. They choose quantities simultaneously and independently and their constant symmetric marginal costs are $c_1 = c_2 = 2$. Inverse demand is given by $P = 14 - Q$, where $Q = q_1 + q_2$.

- a. (6 points) Consider settings where each of the two firms can choose between either colluding and sharing monopoly profits ($\pi^M/2, \pi^M/2$) or competing and receiving Cournot profits. Provide matrix form representation of the simultaneous moves game described above. Identify (graphically) in the (π_1, π_2) -diagram the set of feasible payoffs and the set of sustainable payoffs in this matrix form game. (Hint: Each firm has two possible actions: collude or deviate).
- b. (6 points) Next, assume that the simultaneous moves game in (a) is repeated infinitely many times. Discount factor is δ . Firms follow grim-trigger strategies. Determine graphically the set of Sub-game Perfect Nash Equilibria (SPNEa) of this infinitely repeated game. Determine the smallest value of the discount factor for which collusion on monopoly output constitutes a SPNE.
- c. (4 points) Often in cartel cases lawyers, judges and economic experts state that collusion is more difficult in periods of declining demand than in periods of boom. Can you briefly describe theoretical arguments which support this view?

Question 5: Auctions (16 points)

Consider the *charity auction*. In many charity auctions, altruistic celebrities auction objects with special value for their fans to raise funds for charity. The pop-star *Madonna*, for example, held an auction to sell clothing worn during her career and raised about 3.2 million dollars. In the charity auction the winner of the lot is the highest bidder. The difference with the standard auction is that all bidders are required to pay their bid.

Suppose there are n bidders with valuations randomly drawn from the unit interval, according to the uniform distribution.

- a. (12 points) Derive the equilibrium bidding function without making use of the revenue equivalence theorem.
- b. (2 points) Derive the sellers expected revenue.
- c. (1 points) Without making calculations. Does this auction raise more funds than the standard auctions we have seen in class (first- and second-price sealed bid auctions)? Why?
- d. (1 points) Is this auction efficient from an economic point of view. If not, why? If yes, why?

Question 6: Market Design in Practice (16 points)

A teacher has a classroom with eight pupils, four boys and four girls. In the classroom there are four sets of two tables. The teacher requires that at each set of two tables one boy is sitting next to one girl. The teacher makes the following allocation of pupils over the four sets of tables:

Tables	Man	Woman
1	Adam	Kim
2	Ben	Linda
3	Chris	Mandy
4	Dan	Nicky

Of course, both the boys and the girls have a preference over who they are sitting next to. These preferences are shown in the following table:

Boys	Preferences					Girls	Preferences			
	1	2	3	4			1	2	3	4
Adam	Linda	Kim	Mandy	Nicky		Kim	Dan	Chris	Adam	Ben
Ben	Nicky	Linda	Mandy	Kim		Linda	Chris	Adam	Ben	Dan
Chris	Kim	Linda	Mandy	Nicky		Mandy	Chris	Adam	Dan	Ben
Dan	Nicky	Kim	Mandy	Linda		Nicky	Chris	Dan	Ben	Adam

All three questions below can be answered in one or two sentences. (Of course, a longer answer is allowed.)

- a. (5 points) Is the allocation Pareto efficient? Explain your answer.
- b. (5 points) Is the allocation stable? Explain your answer.
- c. (6 points) Can the existing allocation be the result of applying deferred acceptance? If yes, explain why. If no, explain how the allocation should be modified?