
This exam has 4 pages and 6 exercises. The result will computed as (total number of points plus 10) divided by 10.
Answers may be given in either English or Dutch.

Exercise 1 (Sets) *(6 + 8 + 8 punten)*

1. Given are three sets A, B, and C with 7, 8, and 9 elements, respectively, in a universe U of 20 elements. Furthermore, we are given that

$$\#(A \cap B) = \#(B \cap C) = \#(C \cap A) = 3,$$
$$\#(A \cap B \cap C) = 1.$$

   Determine the following numbers with the use of Venn diagrams:

$$\#(A \cup B \cup C), \quad \#(A' \cap B' \cap C'), \quad \#(A \cup B)'.$$

2. Determine the Venn diagram for each of the two given formulas and clearly depict which area is covered by the formulas. Are the two sets equal?

$$\big(A \setminus (B \cap C)\big)', \quad A' \cup (B \cap C).$$

3. Check the equality with the use of algebra for sets.

$$(A \cup B)' \cup (A \cap C)' = A' \cap (B \cap C)'.$$

(NB: The laws for the algebra of sets are given on the last page of this exam)

Exercise 2 (Syntax) *(5 + 5 points)*

(a) Draw the parse tree of the formula $\neg\,(p \to \neg q) \to (\neg q \to p)$

(b) Compute in the parse tree bottom-up the truth value of this formula, given the truth values F, T for p, q.

Exercisee 3 (Logic) *(8 + 4 points)*

(a) Investigate validity of the semantical implication:

$$(p \vee q) \to q \;\models\; \neg p$$

(Show clearly how you get to your answer.)

(b) Give a formula ϕ such that $\neg\phi \vDash \phi$ is a valid semantic entailment.

Exercise 4 (Logic) *(7 + 6 points)*

(a) Which of the following formulas are semantically equivalent?

$$p \to \neg q,\;\; (p \to q) \to \neg p,\;\; \neg(p \wedge q)$$

(Show clearly how you get to your answer.)

(b) Using only the propositional variable p, give three formulas: a tautology, a contradiction and a contingent formula.

Exercise 5 (Island puzzle) *(10 points)*

On the island of liars and truth speakers everybody is either a liar or a truth speaker. Truth speakers always speak the truth, liars never. Islander b says:

"I am a thief or a liar."

Is b a thief? (Show clearly how you get to your answer.)

Use in this exercise the propositional variables W_b for "b is a truth speaker" and p for "b is a thief".

Exercise 6 (Relations) *(8 + 7 + 8 points)*

(a) ◇ What is the inverse of the binary relation *IsMarriedTo*?
◇ What is the inverse of the composite relation *IsParentOf* ∘ *IsBrotherOrSisterOf*?
◇ Give a simpler name for *IsParentOf* ∘ *IsBrotherOrSisterOf* ∘ *IsChildOf*.

For the remainder of this question we work with the set $A := \{0, 1, 2\}$ with the following relation:

$$R := \{< 0, 0 >, < 0, 1 >, < 1, 1 >, < 1, 2 >, < 2, 1 >\}.$$

(b) Is the relation reflexive? Symmetric? Anti-symmetric? Transitive? Please provide an argument.

(c) Please write down the set R^{-1} by enumeration?

Algebra for sets

Commutativity:

$$A \cup B = B \cup A$$
$$A \cap B = B \cap A$$

Idempotence:

$$A \cup A = A$$
$$A \cap A = A$$

Associativity:

$$A \cup (B \cup C) = (A \cup B) \cup C$$
$$A \cap (B \cap C) = (A \cap B) \cap C$$

Complement:

$$A \cup A' = U$$
$$A \cap A' = \emptyset$$

Distributivity:

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$
$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

DeMorgan's Laws:

$$(A \cup B)' = A' \cap B'$$
$$(A \cap B)' = A' \cup B'$$

Identities:

$$A \cup U = U \text{ en } A \cup \emptyset = A$$
$$A \cap U = A \text{ en } A \cap \emptyset = \emptyset$$

Involution:

$$(A')' = A$$
