2021

Exam 1 solutions

1. Make a note of any student who doesn't answer and I'll request by email immediately.

2. a) valid
   b) sat
   c) unsat
   d) cons
   e) ~~XXXXXX~~ cons

3. a)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\neg p}^{3}\quad\dfrac{}{p \wedge q}^{2}}{p}\quad\dfrac{}{\neg q}^{3}\quad\dfrac{}{p \wedge q}^{2}}{q}}{\neg p \vee \neg q}^{1}\quad\bot\quad\bot}{\bot}^{3}}{\neg(p \wedge q)}^{2}}{(\neg p \vee \neg q) \rightarrow \neg(p \wedge q)}^{1}$$

b)

$$\dfrac{\dfrac{\dfrac{}{\exists z D(z)}^{1}\quad\dfrac{\dfrac{\dfrac{\forall x \forall y(D(y) \rightarrow E(x))}{\forall y(D(y) \rightarrow E(a))}}{D(z) \rightarrow E(a)}\quad\dfrac{}{D(z)}^{2}}{E(a)}}{\forall x E(x)}^{2}}{\exists z D(z) \rightarrow \forall x E(x)}^{1}$$

c) invalid: p, q, t false

d) invalid: R doesn't hold for anything

4. a) $\exists x \forall y (y \neq x \rightarrow B(x, y))$

b) $S(e) \wedge \exists x (S(x) \wedge B(x, e))$

c) $\forall x (O(e, x) \rightarrow \neg L(x))$

d) $\forall x y z (L(x) \wedge L(y) \wedge L(z) \rightarrow x = y \vee x = z \vee y = z)$

5. a) i) Any finite subset of Δ is cons., so by compactness, ...
ii) Since $M \models \Gamma$ it has finitely many (n) elts.
But $M \models \varphi_{n+1}$ which means it has at least $n+1$ elts.

b) This sentence is not valid. recursively enbl: no R.
So by soundness, cannot prove.

6. a) i) unrec 2) rec

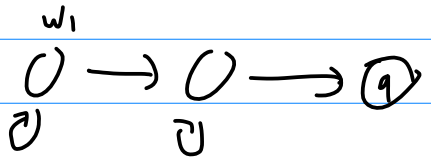
b) This is not a dec. problem - not a yes/no question.

c) The arg. goes: suppose $T(x)$ decides it... construct a program that runs T and does not terminate if T says x term.
But we cannot construct this new prog - it must always term.

7. a) w_1: yes w_2: no w_3: yes

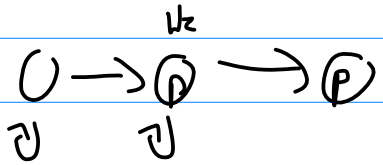
b) w_1: no w_2: no w_3: yes

c) no



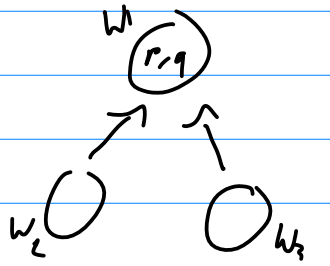
$w_1 \models \Diamond\Diamond q$ but not $\Diamond q$

d) no



$w_2 \models \Box p$ but not $\Box q$

e)



$w_3, w_3 \models \Diamond p \wedge \Diamond q$

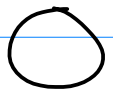
8 a) none. This formula implies no arrows

b) i) not sym, trans

ii) sym, trans

iii) sym. not trans

c)



Model w/ one world, no arrows makes $\Box p$ true but not functional