Vrije Universiteit Amsterdam **April 14, 2022**
Linear Algebra – test 3
Lecturer: Jose Mujica

Instructions:
Please write your answers clearly *on a separate paper.* Write down your name and student number.
Motivate your answers (e.g. indicate the theorems you use, add necessary calculations, etc.).
Use scrap paper to do additional computations if necessary. *No* books/calculators/computers allowed.
You have 40 minutes to complete this test.
Your grade = points +1.

Question 1 (5p)

Consider the matrix

$$A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}.$$

Explain why A is diagonalizable and provide matrices P invertible and D diagonal such that $A = PDP^{-1}$. (*Hint:* $\lambda = -1$ *is an eigenvalue of* A.)

Question 2 (2p)

Let $a, b, c, d \in \mathbb{R}$ and consider the matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$. Show that the characteristic equation of A is given by

$$\lambda^2 - tr(A)\lambda + det(A) = 0.$$

Here, $tr(A) = a + d$ and $det(A) = ad - bc$ are the *trace* and the *determinant* of the matrix A, respectively.

Question 3 (1p,1p)

Decide whether the following statements are *true* or *false*. Explain your answer by giving a proof if the statement is true or giving a proof or providing a counterexample if the statement is false.

(a) Let k be a positive integer. If λ is an eigenvalue of A with with eigenvector $\mathbf{v}$, then λ^k is an eigenvalue of A^k with with the same eigenvector $\mathbf{v}$.

(b) If A is a diagonalizable matrix, then A is invertible.