Vrije Universiteit Amsterdam **March 17, 2022**
Linear Algebra – test 2
Lecturer: Jose Mujica

Instructions:
Please write your answers clearly *on a separate paper*. Write down your name and student number.
Motivate your answers (e.g. indicate the theorems you use, add necessary calculations, etc.).
Use scrap paper to do additional computations if necessary. *No* books/calculators/computers allowed.
You have 40 minutes to complete this test.
Your grade = points +1.

Question 1 (4p)

Find an LU-factorization of the matrix

$$A = \begin{bmatrix} 2 & 4 & -1 & 5 \\ -4 & -5 & 3 & -8 \\ 2 & -5 & -4 & 1 \end{bmatrix},$$

and explain how we can use the LU factorization to solve a matrix equation of the form $A\mathbf{x} = \mathbf{b}$.

Question 2 (1p,1p,1p)

Consider the matrix $A = \begin{bmatrix} k-1 & 1 & 2 \\ 0 & k+1 & 3 \\ k-1 & 1 & k+2 \end{bmatrix}$, with $k \in \mathbb{R}$.

(a) Calculate $det(A)$ in terms of k.

(b) Determine the value(s) of $k \in \mathbb{R}$ such that A is invertible.

(c) For $k = 2$, calculate A^{-1}.

Question 3 (1p,1p)

Decide whether the following statements are *true* or *false*. Explain your answer by giving a proof if the statement is true or giving a proof or providing a counterexample if the statement is false.

(a) If A is a 3×3 matrix, then $det(2A) = 2det(A)$.

(b) If $A = [a_1\ a_2]$ is a 2×2 matrix with columns a_1, a_2 and $det(A) = 0$, then $a_1 = c \cdot a_2$ for some $c \in \mathbb{R}$.