1)

$$A = \begin{bmatrix} 2 & 4 & -1 & 5 \\ -4 & -5 & 3 & -8 \\ 2 & -5 & -4 & 1 \end{bmatrix} \sim \begin{bmatrix} 2 & 4 & -1 & 5 \\ 0 & 3 & 1 & 2 \\ 0 & -9 & -3 & -4 \end{bmatrix}$$

1p For obtaining a correct row-reduced matrix

$$\sim \begin{bmatrix} 2 & 4 & -1 & 5 \\ 0 & 3 & 1 & 2 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

from where $U = \begin{bmatrix} 2 & 4 & -1 & 5 \\ 0 & 3 & 1 & 2 \\ 0 & 0 & 0 & 2 \end{bmatrix}$

1p For identify U. correctly

we obtain L by completing the lower triangular 3x3 matrix with the highlighted columns divided by the corresponding pivots, that is,

$$L = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 1 & -3 & 1 \end{bmatrix}$$

1p For identify L correctly

This gives $A = LU$

To solve $Ax=b$ we use

$A=LU$ and obtain

$$LUx=b.$$

Then define $Ux=y$

and obtain the system $Ly=b$

once we solve $Ly=b$ for y, we

substitute back on $Ux=y$ and

solve for x.

1p For explaining how to use the LU factorization.

2)

(a) We calculate det(A) through row 2, and get

$$\det(A) = k(k+1)(k-1) = k(k^2-1)$$

(1p) For obtainin a correct expression for det(A), not necessarily factorized

(b) if $k = -1, 0, 1$, $\det(A) = 0$. then

A is invertible if $k \neq -1, 0, 1$.

(1p) For the right argument/conclusion.

(c) if $k = 2$, $A = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 3 & 3 \\ 1 & 1 & 4 \end{bmatrix}$

Then

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 2 & 1 & 0 & 0 \\ 0 & 3 & 3 & 0 & 1 & 0 \\ 1 & 1 & 4 & 0 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 3/2 & -1/3 & -1/2 \\ 0 & 1 & 0 & 1/2 & 1/3 & -1/2 \\ 0 & 0 & 1 & -1/2 & 0 & 1/2 \end{array}\right]$$

from where $A^{-1} = \begin{bmatrix} 3/2 & -1/3 & -1/2 \\ 1/2 & 1/3 & -1/2 \\ -1/2 & 0 & 1/2 \end{bmatrix}$

(1p) For obtaining A^{-1} correctly.

3 (a) False, if A is 3×3

$2A$ multiplies each of the three rows of A by 2, so

$$\det(2A) = 2^3 \det(A) = 8\det(A).$$

(1p) For right argument

(b) True, if $A = [a_1\ a_2]$ and $\det(A) = 0$, the columns a_1, a_2 are linearly dependent, then

$c_1 a_1 + c_2 a_2 = 0$ has nontrivial solution c_1, c_2 with, say, $c_1 \neq 0$. Then,

$$a_1 = -\frac{c_2}{c_1} a_2 .$$

Renaming $c = -\frac{c_2}{c_1}$ we obtain the result.

(1p) For right argument.