Vrije Universiteit Amsterdam **March 3rd, 2022**
Linear Algebra – test 1
Lecturer: Jose Mujica

Instructions:
Please write your answers clearly *on a separate paper.* Write down your name and student number.
Motivate your answers (e.g. indicate the theorems you use, add necessary calculations, etc.).
Use scrap paper to do additional computations if necessary. *No* books/calculators/computers allowed.
You have 40 minutes to complete this test.
Your grade = points +1.

Question 1 (3p)

Given the following system of linear equations

$$\begin{cases} x_1 + 2x_2 - 2x_3 + 3x_4 &= 2 \\ 2x_1 + 4x_2 - 3x_3 + 4x_4 &= 5 \\ 5x_1 + 10x_2 - 8x_3 + 11x_4 &= 12 \end{cases}$$

Find the general solution and write it in parametric vector form.

Question 2 (3p)

Consider the matrix $A = \begin{bmatrix} 1 & h \\ 4 & 8 \end{bmatrix}$ and the vector $\underline{\mathbf{b}} = \begin{bmatrix} 2 \\ k \end{bmatrix}$.

Determine the value(s) of $h, k \in \mathbb{R}$ such that the matrix equation $A\underline{\mathbf{x}} = \underline{\mathbf{b}}$ has

(a) a unique solution,

(b) infinitely many solutions,

(c) no solution.

Question 3 (3p)

Decide whether the following statements are *true* or *false.* Explain your answer.

(a) The vector $\underline{\mathbf{u}} = \begin{bmatrix} 0 \\ 4 \\ 4 \end{bmatrix}$ lies in the span of the vectors $\underline{\mathbf{v}} = \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}$ and $\underline{\mathbf{w}} = \begin{bmatrix} -5 \\ 6 \\ 1 \end{bmatrix}$.

(b) There exists a linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ such that
$T\left(\begin{bmatrix} 1 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$, $T\left(\begin{bmatrix} 2 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ and $T\left(\begin{bmatrix} 3 \\ 2 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

(c) If v_1, v_2, v_3, v_4 are linearly independent vectors in $\mathbb{R}^4$, then the set $\{v_1, v_2, v_3\}$ is also linearly independent in $\mathbb{R}^4$.