Vrije Universiteit Amsterdam, Faculty of Science **April 22, 2021**
Linear Algebra test 4
You have 40 minutes to complete this test.
grade = points + 1

Use of calculators, books or notes is not allowed. Motivate your answers.

Question 1 (3p)

Given is the matrix $A = \begin{bmatrix} 5 & 4 \\ -2 & 1 \end{bmatrix}$.

(a) Find the eigenvalues of A.

(b) Give an invertible matrix P, and a matrix C of the form $\begin{bmatrix} a & -b \\ b & a \end{bmatrix}$, such that $AP = PC$.

Question 2 (3p)

Given are the vectors $\mathbf{w_1} = \begin{bmatrix} 2 \\ -1 \\ 2 \\ -1 \end{bmatrix}$, $\mathbf{w_2} = \begin{bmatrix} 1 \\ 2 \\ 1 \\ 2 \end{bmatrix}$, $\mathbf{w_3} = \begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$ and $\mathbf{y} = \begin{bmatrix} 5 \\ 0 \\ 5 \\ 10 \end{bmatrix}$. Let $W = \text{Span}\{\mathbf{w_1}, \mathbf{w_2}, \mathbf{w_3}\}$.

(a) Show that $\{\mathbf{w_1}, \mathbf{w_2}, \mathbf{w_3}\}$ is an orthogonal basis for W.
(b) Find the orthogonal projection of $\mathbf{y}$ onto W.
(c) Compute the distance from $\mathbf{y}$ to W.

Question 3 (3p)

Mark each of the following statements *true* or *false*. If the statement is *true*, give a proof. If the statement is *false*, give a proof or provide a counterexample.

(a) Let $\mathbf{x}$, $\mathbf{y}$ and $\mathbf{z}$ be three distinct vectors in $\mathbb{R}^2$. If $\mathbf{x}$ is orthogonal to $\mathbf{y}$, and $\mathbf{y}$ is orthogonal to $\mathbf{z}$, then $\mathbf{x}$ is orthogonal to $\mathbf{z}$.

(b) If $\{\mathbf{u_1}, \mathbf{u_2}, \mathbf{u_3}\}$ is an orthonormal set of vectors in $\mathbb{R}^n$, then $||\mathbf{u_1} + \mathbf{u_2} + \mathbf{u_3}|| = \sqrt{3}$.

(c) Let λ be a complex eigenvalue of the orthogonal matrix A. Then the modulus of λ is one, i.e. $|\lambda| = 1$.