1a. $\begin{vmatrix} 5-\lambda & 4 \\ -2 & 1-\lambda \end{vmatrix} = (5-\lambda)(1-\lambda) + 8 = 0$

$$\lambda^2 - 6\lambda + 13 = 0$$

$$(\lambda - 3)^2 = -4$$

$$\lambda - 3 = \pm 2i$$

$$\lambda_1 = 3+2i, \quad \lambda_2 = 3-2i$$

1b. $A - (3-2i)I = \begin{bmatrix} 2+2i & 4 \\ -2 & -2+2i \end{bmatrix} \sim \begin{bmatrix} 1+i & 2 \\ 0 & 0 \end{bmatrix}$

$$\longrightarrow \underline{v}_1 = \begin{bmatrix} -2 \\ 1+i \end{bmatrix}$$

$A = PCP^{-1}$ with $P = \begin{bmatrix} -2 & 0 \\ 1 & 1 \end{bmatrix}$ and

$$C = P^{-1}AP = \begin{bmatrix} 3 & -2 \\ 2 & 3 \end{bmatrix}.$$

or: $P = \begin{bmatrix} 2 & 0 \\ -1 & 1 \end{bmatrix}$ and $C = \begin{bmatrix} 3 & 2 \\ -2 & 3 \end{bmatrix}$

$P = \begin{bmatrix} 1 & -1 \\ -1 & 0 \end{bmatrix}$ and $C = \begin{bmatrix} 3 & -2 \\ 2 & 3 \end{bmatrix}$

$P = \begin{bmatrix} 1 & 1 \\ -1 & 0 \end{bmatrix}$ and $C = \begin{bmatrix} 3 & 2 \\ -2 & 3 \end{bmatrix}$

(for any nonzero multiple of the columns of P)

2a. $$\left.\begin{aligned} \underline{w}_1 \cdot \underline{w}_2 &= 0 \\ \underline{w}_1 \cdot \underline{w}_3 &= 0 \\ \underline{w}_2 \cdot \underline{w}_3 &= 0 \end{aligned}\right\}$$ The vectors are orthogonal, and therefore are linearly independent. (They also span W.)

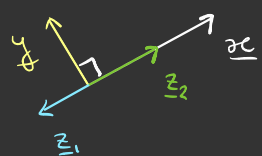
2b. We can use
$$\hat{\underline{y}} = \frac{\underline{y}\cdot\underline{w}_1}{\underline{w}_1\cdot\underline{w}_1}\underline{w}_1 + \frac{\underline{y}\cdot\underline{w}_2}{\underline{w}_2\cdot\underline{w}_2}\underline{w}_2 + \frac{\underline{y}\cdot\underline{w}_3}{\underline{w}_3\cdot\underline{w}_3}\underline{w}_3$$
$$= \frac{10}{10}\underline{w}_1 + \frac{30}{10}\underline{w}_2 + 0\,\underline{w}_3$$
$$= \begin{bmatrix} 2 \\ -1 \\ 2 \\ -1 \end{bmatrix} + \begin{bmatrix} 3 \\ 6 \\ 3 \\ 6 \end{bmatrix} = \begin{bmatrix} 5 \\ 5 \\ 5 \\ 5 \end{bmatrix}.$$

2c. $$\text{dist}(\underline{y}, W) = \|\underline{y} - \hat{\underline{y}}\| = \left\|\begin{bmatrix} 0 \\ -5 \\ 0 \\ 5 \end{bmatrix}\right\| = \sqrt{5^2+5^2} = \sqrt{50} = 5\sqrt{2}.$$

3a. <u>False</u> Counterexample: $\underline{x} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, $\underline{y} = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$, $\underline{z} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$

Then $\underline{x}\cdot\underline{y} = 0$, $\underline{y}\cdot\underline{z} = 0$,
but $\underline{x}\cdot\underline{z} \neq 0$.

Or draw/explain:



$\underline{z}$ can be a nonzero multiple of $\underline{x}$.

3b. True $\quad \|\underline{u}_1 + \underline{u}_2 + \underline{u}_3\|^2 = (\underline{u}_1 + \underline{u}_2 + \underline{u}_3) \cdot (\underline{u}_1 + \underline{u}_2 + \underline{u}_3)$

$= \underline{u}_1 \cdot \underline{u}_1 + \underline{u}_2 \cdot \underline{u}_2 + \underline{u}_3 \cdot \underline{u}_3,$

since $\underline{u}_i \cdot \underline{u}_j = 0$ for $i \neq j$,

$= \|\underline{u}_1\|^2 + \|\underline{u}_2\|^2 + \|\underline{u}_3\|^2$

$= 1 + 1 + 1 = 3.$

So, $\|\underline{u}_1 + \underline{u}_2 + \underline{u}_3\| = \sqrt{3}$. (length is always positive)

3c. True $\quad$ Let $\lambda \in \mathbb{C}$. We know $A\underline{x} = \lambda\underline{x}$, for some nonzero $\underline{x}$. It follows that $\|A\underline{x}\| = \|\lambda \underline{x}\|$. But $\|A\underline{x}\| = \|\underline{x}\|$, since A is orthogonal. So, $\|\underline{x}\| = \|\lambda\underline{x}\|$. Then $\|\underline{x}\| = |\lambda|\,\|\underline{x}\|$, which implies $|\lambda| = 1$, since $\|\underline{x}\| \neq 0$.