Vrije Universiteit Amsterdam, Faculty of Science **April 8, 2021**
Linear Algebra test 3
You have 40 minutes to complete this test.
grade = points + 1

Use of calculators, books or notes is not allowed. Motivate your answers.

Question 1 (5p)

Given is the matrix $A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 2 & -1 & 2 \end{bmatrix}$.

(a) Is A invertible? What does this tell you about the eigenvalues of A?
(b) Show that $\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$ is an eigenvector of A. What is the corresponding eigenvalue?
(c) Find all eigenvalues of A.
(d) Is A diagonalizable? Explain.

Question 2 (2p)

Let $T : \mathbb{R}^2 \to \mathbb{P}_2$ be a linear transformation defined by $T\left(\begin{bmatrix} a \\ b \end{bmatrix}\right) = a + b + (a+b)t^2$.

Find the matrix M for T relative to the bases $\mathcal{B} = \left\{\begin{bmatrix} 0 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ -2 \end{bmatrix}\right\}$ and $\mathcal{C} = \{1, t, t + t^2\}$.

Question 3 (2p)

Mark each of the following two statements *true* or *false*. If the statement is *true*, give a proof. If the statement is *false*, give a proof or provide a counterexample.

(a) If A is invertible and 1 is an eigenvalue of A, then 1 is also an eigenvalue of A^{-1}.

(b) An $n \times n$ matrix with n linearly independent eigenvectors is invertible.