Vrije Universiteit Amsterdam   **Thursday March 4, 2021**
Linear Algebra test 2
You have 40 minutes to complete this test.
grade = points + 1

Use of calculators, books or notes is not allowed. Motivate your answers.

Question 1 (2p)

Give the LU-factorization of the matrix $A = \begin{bmatrix} 2 & 4 & 0 & 0 & 2 \\ 3 & 6 & 4 & 4 & 7 \\ 1 & 2 & 3 & 4 & 5 \end{bmatrix}$.

Question 2 (1.5p,1.5p,1p)

Given is the matrix $B = \begin{bmatrix} 1 & 2 & 0 \\ 5 & k & 3 \\ 4 & 0 & k \end{bmatrix}$ with $k \in \mathbb{R}$.

(a) Compute the determinant of the matrix B in terms of k.
(b) Let $k = 2$. Find the inverse matrix B^{-1}.
(c) For what value(s) of k is the matrix B invertible?

Question 3 (1p,1p,1p)
Mark each of the following statements *true* or *false*. If the statement is true, give a proof. If the statement is false, give a proof or provide a counterexample.

(a) Let A and B be $n \times n$ matrices. If $AB = I$, then $AB = BA$.

(b) Let $T : \mathbb{R}^n \to \mathbb{R}^n$ be a linear transformation. If T maps $\mathbb{R}^n$ onto $\mathbb{R}^n$, then the inverse transformation T^{-1} exists and is one-to-one.

(c) If A is a 20×20 matrix, then $\det(-A) = -\det(A)$.