1. $A = \begin{bmatrix} 2 & 4 & 0 & 0 & 2 \\ 3 & 6 & 4 & 4 & 7 \\ 1 & 2 & 3 & 4 & 5 \end{bmatrix} \sim \begin{bmatrix} 2 & 4 & 0 & 0 & 2 \\ 0 & 0 & 4 & 4 & 4 \\ 0 & 0 & 3 & 4 & 4 \end{bmatrix}$

$$\sim \begin{bmatrix} 2 & 4 & 0 & 0 & 2 \\ 0 & 0 & 4 & 4 & 4 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} = U$$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 3/2 & 1 & 0 \\ 1/2 & 3/4 & 1 \end{bmatrix}$$

2a. $\det B = \begin{vmatrix} 1 & 2 & 0 \\ 5 & k & 3 \\ 4 & 0 & k \end{vmatrix} = 1 \cdot \begin{vmatrix} k & 3 \\ 0 & k \end{vmatrix} - 2 \cdot \begin{vmatrix} 5 & 3 \\ 4 & k \end{vmatrix}$

$$= k^2 - 10k + 24$$

2b. $\left[\begin{array}{ccc|ccc} 1 & 2 & 0 & 1 & 0 & 0 \\ 5 & 2 & 3 & 0 & 1 & 0 \\ 4 & 0 & 2 & 0 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1 & 2 & 0 & 1 & 0 & 0 \\ 0 & -8 & 3 & -5 & 1 & 0 \\ 0 & -8 & 2 & -4 & 0 & 1 \end{array}\right]$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 2 & 0 & 1 & 0 & 0 \\ 0 & -8 & 3 & -5 & 1 & 0 \\ 0 & 0 & -1 & 1 & -1 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1 & 2 & 0 & 1 & 0 & 0 \\ 0 & -8 & 0 & -2 & -2 & 3 \\ 0 & 0 & 1 & -1 & 1 & -1 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/2 & -1/2 & 3/4 \\ 0 & -2 & 0 & -1/2 & -1/2 & 3/4 \\ 0 & 0 & 1 & -1 & 1 & -1 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/2 & -1/2 & 3/4 \\ 0 & 1 & 0 & 1/4 & 1/4 & -3/8 \\ 0 & 0 & 1 & -1 & 1 & -1 \end{array}\right]$$

$$= B^{-1}$$

2c. B is invertible if $\det B \neq 0$.

$\det B = k^2 - 10k + 24 = (k-4)(k-6) \neq 0$,

so for $k \neq 4$ and $k \neq 6$.

3a. <u>True</u> Let A and B be $n \times n$ matrices. Suppose that $AB = I$.
By the invertible matrix theorem, this implies that A and B are invertible.
It follows that $(AB)B^{-1} = IB^{-1}$, so $A = B^{-1}$. Therefore, $BA = BB^{-1} = I = AB$.

3b. <u>True</u> Let $T: \mathbb{R}^n \to \mathbb{R}^n$ be a linear transformation. Let A be the standard matrix of T. Suppose T is onto.
By the invertible matrix theorem, this implies that A is invertible. Therefore, the inverse transformation T^{-1} exists and has standard matrix A^{-1}. Since A^{-1} is invertible, T^{-1} is one-to-one, by the invertible matrix theorem.

3c. False The matrix $-A$ is formed by multiplying every row of A with -1. When one row of A is multiplied by -1, then the determinant of A is multiplied by -1. Since A has 20 rows, $\det(-A) = (-1)^{20} \det(A)$ $= \det A$.