

Use of calculators, books or notes is not allowed. Motivate your answers.

Question 1 (2.5p)

Given is the following system of linear equations

$$2x_1 + 4x_2 + 8x_3 = -2$$

$$x_1 + x_2 + x_3 = 1$$

$$4x_1 + 6x_2 + 10x_3 = 0$$

Find the general solution and write the solution in parametric vector form.

Question 2 (3.5p)

Given are the matrix $A = \begin{bmatrix} -1 & 5 & -3 \\ -1 & -3 & 1 \\ 4 & 0 & h \\ 0 & 2 & -1 \end{bmatrix}$ and the vector $\mathbf{b} = \begin{bmatrix} 2 \\ h \\ 2 \\ 1 \end{bmatrix}$.

Determine the value(s) of h (if any) for which

- (a) the matrix equation $A\mathbf{x} = \mathbf{b}$ is inconsistent;
- (b) the matrix equation $A\mathbf{x} = \mathbf{b}$ has infinitely many solutions;
- (c) the columns of A are linearly independent;
- (d) the columns of A span \mathbb{R}^4 .

Question 3 (3p)

Mark each of the following statements *true* or *false*. If the statement is *true*, give a proof. If the statement is *false*, give a proof or provide a counterexample.

(a) If an $m \times n$ matrix A with $m > n$ has a pivot position in every column, then the equation $A\mathbf{x} = \mathbf{b}$ is consistent for every \mathbf{b} in \mathbb{R}^m .

(b) The transformation $T : \mathbb{R}^2 \rightarrow \mathbb{R}^3$ defined by $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x + y \\ 0 \\ xy \end{bmatrix}$ is a linear transformation.