Faculty of Sciences	Testexam Linear Algebra
Vrije Universiteit Amsterdam	Wednesday, May 27, 2020 (18:30-19:30)

Please read the following instructions carefully: Use of a basic calculator is allowed. Please provide an argument or calculation at every question. When you are finished, make photos of your solutions and upload them as **a single pdf file** to Canvas. Make sure the photo is clearly readable. Save your original solutions until your grade is determined.

Question 1 [8 pnt]. Let

$$A = \begin{bmatrix} 3 & 5 & -4 \\ -3 & -2 & 4 \\ 6 & 1 & -8 \end{bmatrix}.$$

a) [3 pnt] Determine a basis for the null space of A.

b) [3 pnt] Determine a basis for the column space of A.

The matrix A can be viewed as a linear transformation $T_A : \mathbb{R}^3 \to \mathbb{R}^3$.

c) [1 pnt] Is this transformation one-to-one? *Hint:* Use your answer to a).

d) [1 pnt] Is this transformation onto $\mathbb{R}^3$? *Hint:* Use your answer to b).

Question 2 [6 pnt] Let $u = \begin{bmatrix} 2/3 \\ 2/3 \\ 1/3 \end{bmatrix}$, $x = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, and $B = uu^T$.

a) [2 pnt] Compute Bx and show that it equals the orthogonal projection of x onto u.

b) [2 pnt] Show that B is symmetric and that $B^2 = B$.

c) [2 pnt] Show that u is an eigenvector of B and determine its eigenvalue.

Question 3 [5 pnt] Determine if the following statements are true or false. Provide an argument for your answer.

a) [1 pnt] If the system $Ax = b$ is inconsistent, then b is not in the set spanned by the columns of A.

b) [1 pnt] An eigenspace of a matrix A is the null space of a certain matrix.

c) [1 pnt] If x is not in a subspace W, then $x - \text{proj}_W x$ is not zero.

d) [1 pnt] If A is a 3×2 matrix, then the transformation $x \mapsto Ax$ cannot be onto $\mathbb{R}^3$.

e) [1 pnt] An $n \times n$ symmetric matrix has n distinct real eigenvalues.