Vrije Universiteit Amsterdam, Faculty of Science **May 1, 2020**
Linear Algebra test 4
You have 45 minutes to complete this test. You have to upload your answers to Canvas before 10.40.
grade = points + 1

Use of calculators, books or notes is not allowed. Motivate your answers.

Question 1 (1p,1p,2p)

Given are the matrix $A = \begin{bmatrix} 1 & 2 \\ 1 & 1 \\ -1 & 0 \end{bmatrix}$ and the vector $\mathbf{b} = \begin{bmatrix} 4 \\ 3 \\ 4 \end{bmatrix}$.

(a) Find an orthogonal basis for the column space of A.
(b) Find a QR factorization of A.
(c) The vector $\mathbf{b}$ can be written as $\mathbf{b} = \mathbf{b}_1 + \mathbf{b}_2$ with $\mathbf{b}_1 \in \text{Col } A$ and $\mathbf{b}_2 \in (\text{Col } A)^\perp$. Find $\mathbf{b}_1$ and $\mathbf{b}_2$.

Question 2 (1p,1p,1p)

Consider the inner product space $\mathbb{P}_2$ of all polynomials of degree less than or equal to 2, with inner product

$$\langle \mathbf{p}, \mathbf{q} \rangle = \mathbf{p}(1)\mathbf{q}(1) + \mathbf{p}(2)\mathbf{q}(2) + \mathbf{p}(3)\mathbf{q}(3).$$

(a) Explain why $\langle \mathbf{p}, \mathbf{p} \rangle = 0$ if and only if $\mathbf{p}(t) = 0$ for all $t \in \mathbb{R}$.

Let $\mathbf{p}(t) = 3 - t$ and $\mathbf{q}(t) = 4t - 2t^2$.

(b) Compute the length of $\mathbf{p}$.

(c) Find the orthogonal projection of $\mathbf{q}$ onto the subspace spanned by $\mathbf{p}$.

Question 3 (1p,1p)

Mark each of the following two statements *true* or *false*. If the statement is *true*, give a proof. If the statement is *false*, give a proof or provide a counterexample.

(a) Let $\mathbf{x}$, $\mathbf{y}$ and $\mathbf{z}$ be vectors in $\mathbb{R}^n$. If $\mathbf{x}$ and $\mathbf{y}$ are orthogonal, and $\mathbf{y}$ and $\mathbf{z}$ are orthogonal, then $\mathbf{x}$ and $\mathbf{z}$ are orthogonal.

(b) Let U be a 2×2 orthogonal matrix. If $\{\mathbf{v}_1, \mathbf{v}_2\}$ is an orthonormal set in $\mathbb{R}^2$, then so is $\{U\mathbf{v}_1, U\mathbf{v}_2\}$.