Vrije Universiteit Amsterdam, Faculty of Science **April 17, 2020**
Linear Algebra test 3
You have to upload your answers on Canvas before 11.00.
grade = points + 1

Use of calculators, books or notes is not allowed. Motivate your answers.

Question 1 (0.5p,1p)

Given is the matrix $A = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 2 \\ 1 & 1 & 1 \end{bmatrix}$.

(a) Compute the eigenvalues of A.
(b) Is A diagonalizable? Explain.

Question 2 (1p,0.5p,1p,0.5p)

(a) Let B be an $n \times n$ matrix with eigenvalue λ. Explain why the echelon form of $B - \lambda I$ must contain at least one zero row.

Given is the matrix $B = \begin{bmatrix} 4 & -5 \\ 10 & 6 \end{bmatrix}$.
(b) Use the characteristic equation of B to show that the eigenvalues of B are $\lambda_1 = 5 + 7i$ and $\lambda_2 = 5 - 7i$.

We now want to find the corresponding eigenspaces. For λ_1 we compute $B - \lambda_1 I = \begin{bmatrix} -1-7i & -5 \\ 10 & 1-7i \end{bmatrix}$.
By part (a), we can choose to row reduce this matrix to $\begin{bmatrix} -1-7i & -5 \\ 0 & 0 \end{bmatrix}$, or to $\begin{bmatrix} 0 & 0 \\ 10 & 1-7i \end{bmatrix}$.
(c) Show that both approaches result in the same eigenspace for λ_1.
(d) Knowing the eigenspace for λ_1, how can we easily find the eigenspace for λ_2?

Question 3 (1p,1p)

Mark each of the following two statements *true* or *false*. If the statement is *true*, give a proof. If the statement is *false*, give a proof or provide a counterexample.

(a) Let A be an $n \times n$ matrix. If v_1 and v_2 are eigenvectors of A, then $v_1 + v_2$ is an eigenvector of A.

(b) Let A and B be similar matrices. If A is diagonalizable, then B is diagonalizable.

-TEST CONTINUES ON THE BACK-

Question 4 (0.5p,1p,1p)

Consider the discrete dynamical system described by the difference equation $\mathbf{x}_{k+1} = M\mathbf{x}_k$ with

$M = \begin{bmatrix} 1.8 & -0.6 \\ 0.8 & 0.2 \end{bmatrix}$.

(a) Show that $\mathbf{v}_1 = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$ and $\mathbf{v}_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ are eigenvectors of M.

(b) Let $\mathbf{x}_0 = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$ be the initial state of the system. Find an explicit formula for $\mathbf{x}_k$ in terms of k and the eigenvalues and eigenvectors of M, and describe what happens to $\mathbf{x}_k$ as $k \to \infty$.

(c) Same question, but now with initial state $\mathbf{x}_0 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$.

[BONUS] Draw some trajectories ($\mathbf{x}_0$ - $\mathbf{x}_1$ - $\mathbf{x}_2$ - ...) of this dynamical system in the x, y-plane.