Vrije Universiteit Amsterdam **Friday March 6, 2020**
Linear Algebra test 2
You have 40 minutes to complete this test.
grade = points + 1

Use of calculators, books or notes is not allowed. Motivate your answers.

Question 1 (2p)

Give the LU-factorization of the matrix $A = \begin{bmatrix} 2 & 1 & 2 & -1 \\ 1 & 2 & 2 & 1 \\ -1 & 1 & -1 & 1 \end{bmatrix}$.

Question 2 (1.5p)

Suppose $AB = \begin{bmatrix} 8 & 7 \\ 6 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix}$. Find the matrix A.

Question 3 (1.5p,1p)

Given is the matrix $C = \begin{bmatrix} 2 & 8 & -1 \\ -1 & k & 0 \\ 0 & 6 & k \end{bmatrix}$ with $k \in \mathbb{R}$.

(a) Compute the determinant of the matrix C in terms of k.
(b) For what value(s) of k is the matrix C invertible?

Question 4 (1p,1p,1p)

Mark each of the following two statements *true* or *false*. If the statement is true, give a proof. If the statement is false, give a proof or provide a counterexample.

(a) Let A be an $n \times n$ matrix. If every vector $\mathbf{b}$ in $\mathbb{R}^n$ is a linear combination of the columns of A, then the matrix A is invertible.

(b) The linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ defined by $T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - x_2 \\ 2x_1 + 3x_2 \end{bmatrix}$ is invertible.

(c) Let A and B be matrices such that AB is defined. If $AB = 0$, then $A = 0$ or $B = 0$.