Linear Algebra test 2, 2020

1) $$A = \begin{bmatrix} \boxed{2} & 1 & 2 & -1 \\ 1 & 2 & 2 & 1 \\ -1 & 1 & -1 & 1 \end{bmatrix} \sim \begin{bmatrix} 2 & 1 & 2 & -1 \\ 0 & \boxed{3/2} & 1 & 3/2 \\ 0 & 3/2 & 0 & 1/2 \end{bmatrix} \sim \begin{bmatrix} 2 & 1 & 2 & -1 \\ 0 & 3/2 & 1 & 3/2 \\ 0 & 0 & \boxed{-1} & -1 \end{bmatrix} = U.$$ 1

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 1/2 & 1 & 0 \\ -1/2 & 1 & 1 \end{bmatrix}$$ 1

do not give this point if row interchanges or scaling are used.

2) ($\det B \neq 0$, so B is invertible.)

$ABB^{-1} = A$. 1/2

$$B^{-1} = \frac{1}{4-6}\begin{bmatrix} 1 & -3 \\ -2 & 4 \end{bmatrix} = -\frac{1}{2}\begin{bmatrix} 1 & -3 \\ -2 & 4 \end{bmatrix}$$ $\frac{1}{2}$

$$(AB)B^{-1} = \begin{bmatrix} 8 & 7 \\ 6 & 5 \end{bmatrix}\begin{bmatrix} 1 & -3 \\ -2 & 4 \end{bmatrix} \cdot -\frac{1}{2} = \begin{bmatrix} -6 & 4 \\ -4 & 2 \end{bmatrix} \cdot -\frac{1}{2} = \begin{bmatrix} 3 & -2 \\ 2 & -1 \end{bmatrix} = A.$$ $\frac{1}{2}$

3a) $$\det C = \begin{vmatrix} 2 & 8 & -1 \\ -1 & k & 0 \\ 0 & 6 & k \end{vmatrix} = 2\begin{vmatrix} k & 0 \\ 6 & k \end{vmatrix} + 1 \cdot \begin{vmatrix} 8 & -1 \\ 6 & k \end{vmatrix} = 2k^2 + 8k + 6.$$ 1 $\frac{1}{2}$

3b) invertible $\Leftrightarrow \det C \neq 0$ $\frac{1}{2}$

$2k^2 + 8k + 6 = 0$

$k^2 + 4k + 3 = 0$

$(k+3)(k+1) = 0$

C is invertible for $k \neq 3$ and $k \neq -1$. $\frac{1}{2}$

4a) True If every $b \in \mathbb{R}^n$ is a linear combination of the columns of A, then $Ax = b$ is consistent for every $b \in \mathbb{R}^n$ $\frac{1}{2}$ (alternatively: "then the columns of A span $\mathbb{R}^n$"). By the IMT (A is square), the matrix A is invertible. $\frac{1}{2}$

4b) True The standard matrix is $A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$ $\frac{1}{2}$ and A is invertible, (for A clearly has two pivots) so T is invertible $\frac{1}{2}$

4c) False Counterexample: 1

let $A = \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$. Then $AB = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$.