Vrije Universiteit Amsterdam **Friday February 21, 2020**
Linear Algebra test 1
You have 40 minutes to complete this test.
grade = points + 1

Use of calculators, books or notes is not allowed. Motivate your answers.

Question 1 (2p)
Given is the following system of linear equations

$$\begin{aligned} 3x_1 + \ \ 8x_2 - 7x_3 - 5x_4 &= 9 \\ 6x_2 - 3x_3 - 6x_4 &= 9 \\ -3x_1 - 12x_2 + 9x_3 + 9x_4 &= -15 \end{aligned}$$

Find the general solution and write the solution in parametric vector form.

Question 2 (1p,1p,1p,1p)
Consider the vectors $\mathbf{v}_1 = \begin{bmatrix} 1 \\ -2 \\ 1 \\ 2 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 2 \\ -1 \\ 3 \\ 1 \end{bmatrix}$, $\mathbf{v}_3 = \begin{bmatrix} 1 \\ 4 \\ k \\ -4 \end{bmatrix}$ and $\mathbf{y} = \begin{bmatrix} 3 \\ 3 \\ -2 \\ -3 \end{bmatrix}$.

Determine the value(s) of k such that
(a) the vector equation $x_1\mathbf{v}_1 + x_2\mathbf{v}_2 + x_3\mathbf{v}_3 = \mathbf{y}$ is inconsistent;
(b) the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is linearly independent;
(c) $\mathbf{y}$ is in Span$\{\mathbf{v}_2, \mathbf{v}_3\}$;
(d) the vectors $\mathbf{v}_1$, $\mathbf{v}_2$ and $\mathbf{v}_3$ span $\mathbb{R}^4$.

Question 3 (1p,1p,1p)
Mark each of the following two statements *true* or *false*. If the statement is true, give a proof. If the statement is false, give a proof or provide a counterexample.

(a) Let A be an $m \times n$ matrix with $m > n$, and let $\mathbf{b}$ and $\mathbf{c}$ be vectors in $\mathbb{R}^m$. If $A\mathbf{x} = \mathbf{b}$ has a unique solution, then $A\mathbf{x} = \mathbf{c}$ also has a unique solution.

(b) Let A be an $m \times n$ matrix. If $m < n$, then the columns of A are linearly dependent.

(c) The transformation $T : \mathbb{R}^2 \to \mathbb{R}^3$ defined by $T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ 1 - x_2 \\ 2x_1 \end{bmatrix}$ is a linear transformation.