



Faculty of Economics and Business Administration

Exam: Investments 3.4

Code: E\_BE3\_INV

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Date: May 17, 2016

Time: 8.45

Duration: 2 hours and 45 minutes

Calculator allowed: Yes

Graphical calculator  
allowed: Yes

Number of questions: 30 multiple choice questions and 4 open-ended questions

Type of questions: Open questions / multiple choice questions

Answer in: English

Remarks: Be concise and complete in your answers (including calculations). Always explain the answers to the open questions, even if not explicitly called for. Use your time efficiently, using the maximum number of points per question as a guideline

Credit score: You can receive between 0 and 9 points for the exam.

Grades: The grades will be made public on: Wednesday, May 25 2016.

Inspection: Friday, May 27 2016 at 09.00. Room number: to be announced on bb.

Number of pages: 18 (including front page)

**Good luck!**

**PART 1 (MULTIPLE CHOICE QUESTIONS; 30 questions providing 5 points at maximum)**

**Read the questions and answers carefully and write down your answer on your answer sheet. Your final score is determined as (number of correct answers - 4)\*5/26. Negative scores for this part of the exam are set to zero.**

1. The Capital Allocation Line can be described as the
- A. investment opportunity set formed with a risky asset and a risk-free asset.
  - B. investment opportunity set formed with two risky assets.
  - C. line on which lie all portfolios that offer the same utility to a particular investor.
  - D. line on which lie all portfolios with the same expected rate of return and different standard deviations.
  - E. investment opportunity set formed with multiple risky assets.

The CAL has an intercept equal to the risk-free rate. It is a straight line through the point representing the risk-free asset and the risky portfolio, in expected-return/standard deviation space.

2. The utility score an investor assigns to a particular portfolio, other things equal,
- A. will decrease as the rate of return increases.
  - B. will decrease as the standard deviation decreases.
  - C. will decrease as the variance decreases.
  - D. will increase as the variance increases.
  - E. will increase as the rate of return increases.

Utility is enhanced by higher expected returns and diminished by higher risk.

3. Security X has expected return of 14% and standard deviation of 22%. Security Y has expected return of 16% and standard deviation of 28%. If the two securities have a correlation coefficient of 0.8, what is their covariance?
- A. 0.038
  - B. 0.049
  - C. 0.018
  - D. 0.013
  - E. 0.054

$$\text{Cov}(r_x, r_y) = (.8)(.22)(.28) = .04928$$

4. Consider the single-index model. The alpha of a stock is 0%. The return on the market index is 10%. The risk-free rate of return is 5%. The stock earns a return that exceeds the risk-free rate by 5% and there are no firm-specific events affecting the stock performance. The  $\beta$  of the stock is \_\_\_\_\_.

- A. 0.67
- B. 0.75
- C. 1.0**
- D. 1.33
- E. 1.50

$5\% = 0\% + b(5\%); b = 1.0.$

5. An overpriced security will plot

- A. on the Security Market Line.
- B. below the Security Market Line.**
- C. above the Security Market Line.
- D. either above or below the Security Market Line depending on its covariance with the market.
- E. either above or below the Security Market Line depending on its standard deviation.

An overpriced security will have a lower expected return than the SML would predict; therefore it will plot below the SML.

6. According to the CAPM, the risk premium a student taking Investments 3.4 expects to receive on any stock or portfolio increases:

- A. inversely with beta.
- B. inversely with alpha.
- C. positively with beta.**
- D. positively with his/her grade in Investments 3.4.
- E. in proportion to the stock's (portfolio's) standard deviation.

The market rewards systematic risk, which is measured by beta, and thus, the risk premium on a stock or portfolio varies directly with beta.

7. In a well-diversified portfolio

- A. market risk is negligible.
- B. systematic risk is negligible.
- C. unsystematic risk is negligible.**
- D. nondiversifiable risk is negligible.
- E. risk does not exist.

Market, systematic, or nondiversifiable, risk is present in a diversified portfolio; the unsystematic risk has been eliminated.

8. In the APT model, what is the nonsystematic standard deviation of an equally-weighted portfolio that has an average value of  $\sigma(e_i)$  equal to 25% and 50 securities?

- A. 12.5%
- B. 625%
- C. 0.5%
- D. 3.54%**
- E. 14.59%

$$\sigma^2(e_p) = \frac{1}{n} \sigma^2(e_i) = \frac{1}{50} (25)^2 = 12.5, \quad \sigma(e_p) = \sqrt{12.5} = 3.54\%$$

9. Company XYZ just announced yesterday that its first quarter sales were 35% higher than last year's first quarter. You observe that XYZ had an abnormal return of -2% yesterday. This suggests that

- A. the market is not efficient.
- B. XYZ stock will probably rise in value tomorrow.
- C. investors expected the sales increase to be larger than what was actually announced.**
- D. investors expected the sales increase to be smaller than what was actually announced.
- E. earnings are expected to decrease next quarter.

The negative abnormal return suggests that investors expected the sales increase to be larger than what was actually announced.

10. According to proponents of the efficient market hypothesis, the best strategy for a small investor with a portfolio worth €10,000 is probably to

- A. perform fundamental analysis.
- B. exploit market anomalies.
- C. keep his/her money under the mattress.
- D. invest in derivative securities.
- E. invest in mutual funds.**

Individual investors tend to have relatively small portfolios and are usually unable to realize economies of size. The best strategy is to pool funds with other small investors and allow professional managers to invest the funds.

11. An example of \_\_\_\_\_ is that a person may reject an investment when it is posed in terms of risk surrounding potential gains but may accept the same investment if it is posed in terms of risk surrounding potential losses.

- A. framing
- B. regret avoidance
- C. overconfidence
- D. conservatism
- E. None of these is correct.

12. Tests of multifactor models indicate that

- A. the single-factor model has better explanatory power in estimating security returns.
- B. macroeconomic variables have no explanatory power in estimating security returns.
- C. it may be possible to hedge some economic factors that affect future consumption risk with appropriate portfolios.
- D. multifactor models do not work.
- E. None of these is correct.

Tests of multifactor models suggest that industrial production, the risk premium on bonds and unanticipated inflation have significant explanatory power for security returns and it may be possible to hedge these risks if appropriate hedge portfolios can be constructed.

13. Consider the regression equation:  $r_{it} - r_{ft} = g_0 + g_1 b_i + g_2 s^2(e_i) + e_{it}$  where:  $r_{it} - r_{ft}$  = the average difference between the monthly return on stock  $i$  and the monthly risk-free rate;  $b_i$  = the beta of stock  $i$ ;  $s^2(e_i)$  = a measure of the nonsystematic variance of the stock  $i$ . If you estimated this regression equation and the CAPM was valid, you would expect the estimated coefficient  $g_2$  to be

- A. 0.
- B. 1.
- C. equal to the risk-free rate of return.
- D. equal to the average difference between the monthly return on the market portfolio and the monthly risk-free rate.
- E. 3.14.

If the CAPM is valid, the excess return on the stock is predicted by the systematic risk of the stock and the excess return on the market, not by the nonsystematic risk of the stock.

14. If an investor has a portfolio that has constant proportions in T-bills and the market portfolio, the portfolio's characteristic line will plot as a line with \_\_\_\_\_; if the investor can time bull markets, the characteristic line will plot as a line with \_\_\_\_\_.

- A. a positive slope; a negative slope
- B. a negative slope; a positive slope
- C. a constant slope; a negative slope
- D. a negative slope; a constant slope
- E. a constant slope; a positive slope

These characteristics are shown in Figure 24.5. If the proportions are constant the beta of the portfolio stays constant. If the investor switches the proportions in favor of the market portfolio to take advantage of bull markets the beta will increase during times of higher market risk premiums. This will cause the slope of the curve to increase.

15. Hedge fund performance may reflect significant compensation for \_\_\_\_\_ risk.

- A. liquidity
- B. systematic
- C. unsystematic
- D. idiosyncratic
- E. human capital

Hedge fund performance may reflect significant compensation for liquidity risk – recall the LTCM example

16. A zero-coupon bond has a yield to maturity of 11% and a par value of \$1,000. If the bond matures in 27 years, the bond should sell for a price of \_\_\_\_\_ today.

- A. \$59.74
- B. \$501.87
- C. \$513.16
- D. \$483.49
- E. None of these is correct.

$$\text{\$1,000}/(1.11)^{27} = \text{\$59.74}$$

17. What is the relationship between the price of a straight bond and the price of a callable bond?

- A. The straight bond's price will be higher than the callable bond's price for low interest rates.
- B. The straight bond's price will be lower than the callable bond's price for low interest rates.
- C. The straight bond's price will change as interest rates change, but the callable bond's price will stay the same.
- D. The straight bond and the callable bond will have the same price.
- E. There is no consistent relationship between the two types of bonds.

For low interest rates, the price difference is due to the value of the firm's option to call the bond at the call price. The firm is more likely to call the issue at low interest rates, so the option is valuable. At higher interest rates the firm is less likely to call and this option loses value. The prices converge for high interest rates. A graphical representation is shown in Figure 14.4.

18. Forward rates \_\_\_\_\_ future short rates because \_\_\_\_\_.

- A. are equal to; they are both extracted from yields to maturity.
- B. are equal to; they are perfect forecasts.
- C. differ from; they are imperfect forecasts.
- D. differ from; forward rates are estimated from dealer quotes while future short rates are extracted from yields to maturity.
- E. are equal to; although they are estimated from different sources they both are used by traders to make purchase decisions.

Forward rates are the estimates of future short rates extracted from yields to maturity but they are not perfect forecasts because the future cannot be predicted with certainty; therefore they will usually differ.

19. According to the expectations hypothesis, an upward sloping yield curve implies that

- A. interest rates are expected to remain stable in the future.
- B. interest rates are expected to decline in the future.
- C. interest rates are expected to increase in the future.
- D. interest rates are expected to decline first, then increase.
- E. interest rates are expected to increase first, then decrease.

An upward sloping yield curve is based on the expectation that short-term interest rates will increase.

20. The duration of a bond normally increases with an increase in

- A. term to maturity.
- B. yield to maturity.
- C. coupon rate.
- D. All of these are correct.
- E. None of these is correct.

The relationship between duration and term to maturity is a direct one; the relationship between duration and yield to maturity and to coupon rate is negative.

21. Holding other factors constant, the interest-rate risk of a coupon bond is lower when the bond's:

- A. term-to-maturity is higher.
- B. coupon rate is lower.
- C. yield to maturity is higher.
- D. term-to-maturity is higher and coupon rate is lower.
- E. All of these are correct.

The longer the maturity, the greater the interest-rate risk. The lower the coupon rate, the greater the interest-rate risk. The lower the yield to maturity, the greater the interest-rate risk. These concepts are reflected in the duration rules; duration is a measure of bond price sensitivity to interest rate changes (interest-rate risk).

22. Holding other factors constant, which one of the following bonds has the smallest price volatility?

- A. 20-year, 0% coupon bond
- B. 20-year, 6% coupon bond
- C. 20 year, 7% coupon bond
- D. 20-year, 9% coupon bond
- E. Cannot tell from the information given.

Duration (and thus price volatility) is lower when the coupon rates are higher.

23. You took a short position in three S&P 500 futures contracts at a price of 900 (the contract multiplier is 250) and closed the position when the index futures was 885, you incurred:

- A. a gain of \$11,250.
- B. a loss of \$11,250.
- C. a loss of \$3750.
- D. a gain of \$3750.
- E. None of these is correct.

$(\$900 - \$885) = \$15 \times 250 \times 3 = \$11,250.$

24. One reason swaps are desirable is that

- A. they are free of credit risk.
- B. they have no transactions costs.
- C. they increase interest rate volatility.
- D. they increase interest rate risk.
- E. they offer participants easy ways to restructure their balance sheets.

For example, a firm can change a floating-rate obligation into a fixed-rate obligation and vice versa.



25. Before expiration, the time value of a call option is equal to

- A. zero.
- B. the actual call price minus the intrinsic value of the call.**
- C. the intrinsic value of the call.
- D. the actual call price plus the intrinsic value of the call.
- E. None of these is correct.

The difference between the actual call price and the intrinsic value is the time value of the option, which should not be confused with the time value of money. The option's time value is the difference between the option's price and the value of the option were the option expiring immediately.

26. To the option holder, put options are worth \_\_\_\_\_ when the exercise price is higher; call options are worth \_\_\_\_\_ when the exercise price is higher.

- A. more; more
- B. more; less**
- C. less; more
- D. less; less
- E. It doesn't matter - they are too risky to be included in a reasonable person's portfolio.

The holder of the put would prefer to sell the asset to the writer at a higher exercise price. The holder of the call would prefer to buy the asset from the writer at a lower exercise price.

27. The hedge ratio of an option is also called the options \_\_\_\_\_.

- A. alpha
- B. beta
- C. sigma
- D. delta**
- E. rho

The two terms mean the same thing.

28. The buyer of an American call option on a non-dividend paying stock will

- A. always exercise the call as soon as it is in the money.
- B. only exercise the call when the stock price exceeds the previous high.
- C. never exercise the call early.**
- D. buy an offsetting put whenever the stock price drops below the strike price.
- E. None of these is correct.

An American call option buyer will not exercise early if the stock does not pay dividends; exercising forfeits the time value. Rather, the option buyer will sell the option to collect both the intrinsic value and the time value.

29. To hedge a short position in Treasury bonds, an investor most likely would

- A. ignore interest rate futures.
- B. buy S&P futures.
- C. buy interest rate futures.
- D. sell Treasury bonds in the spot market.
- E. None of these is correct.

By taking the long position, the hedger is obligated to accept delivery of T-bonds at the contract maturity date for the current futures price, which locks in the sales price for the bonds and guarantees that the total value of the bond-plus-futures position at the maturity date is the futures price.

30. Who guarantees that a futures contract will be fulfilled?

- A. the buyer
- B. the seller
- C. the broker
- D. the clearinghouse
- E. nobody

Once two parties have agreed to enter the transaction, the clearinghouse becomes the buyer and seller of the contract and guarantees its completion.

## **PART 2 (OPEN QUESTIONS; 4 questions providing 4 points at maximum)**

**Read the questions and answers carefully and write down your answer on your answer sheet.**

### **Question 1. Equilibrium Pricing Models (1 point at maximum)**

#### **Part a. (0.3 points)**

Give the formula of the CAPM and explain its notation. What are the assumptions underlying the CAPM? How do they relate to empirical evidence?

The assumptions are:

- (a) The market is composed of many small investors, who are price-takers; i. e., perfect competition. In reality this assumption was fairly realistic until recent years when institutional investors increasingly began to influence the market with their large transactions.
- (b) All investors have the same holding period. Obviously, different investors have different goals, and thus have different holding periods.
- (c) Investments are limited to those that are publicly traded. In addition, it is assumed that investors may borrow or lend any amount at a fixed, risk-free rate. Obviously, investors may purchase assets that are not publicly traded; however, the dollar volume of publicly traded assets is considerable. The assumption that investors can borrow or lend any

amount at a fixed, risk-free rate obviously is false. However, the model can be modified to incorporate different borrowing and lending rates.

- (d) Investors pay no taxes on returns and incur no transaction costs. Obviously, investors do pay taxes and do incur transaction costs.
- (e) All investors are mean-variance efficient. This assumption implies that all investors make decisions based on maximizing returns available at an acceptable risk level; most investors probably make decisions in this manner. However, some investors are pure wealth maximizers (regardless of the risk level); and other investors are so risk averse that avoiding risk is their only goal.
- (f) All investors have homogeneous expectations, meaning that given the same data all investors would process the data in the same manner, resulting in the same risk/return assessments for all investment alternatives.

### Part b. (0.3 points)

Consider the multifactor APT. There are two independent economic factors, F1 and F2. The risk-free rate of return is 2%. The following information is available about two well-diversified portfolios A and B:

Portfolio	beta on F1	beta on F2
A	0.8	0.3
B	0	1

In addition, the following information is available for the two independent risk factors F1 and F2

Factors	Expected return	Variance
F1	5%	0.11
F2	4%	0.08

- i. Assuming no arbitrage opportunities exist, calculate the expected return of the two portfolios A and B

Simply apply the formula for the two portfolios:

$$ER_i = Rf + \beta_{\{F1\}} * (ER_{\{F1\}} - Rf) + \beta_{\{F2\}} * (ER_{\{F2\}} - Rf)$$

You obtain expected returns of 5% and 4% for portfolios A and B

- ii. Give an expression for the variance of a well-diversified portfolio along the lines of the multifactor APT. Calculate the variance of the two portfolios, given the data above.

Because A and B are well-diversified:

$$\begin{aligned}\beta_{A1}^2 \sigma_{F1}^2 + \beta_{A2}^2 \sigma_{F2}^2 &= \sigma_A^2 \\ \beta_{B1}^2 \sigma_{F1}^2 + \beta_{B2}^2 \sigma_{F2}^2 &= \sigma_B^2\end{aligned}$$

Apply the formula and obtain  $\sigma_A^2 = 0.078$  and  $\sigma_B^2 = 0.08$

- iii. Construct a portfolio of A and B that has exposure of 0.5 to  $F_2$ . What are the weights of the two portfolios A and B? What is the exposure to  $F_1$  of the newly created portfolio?

The weights of A and B should satisfy the following system of equations:

$$\begin{aligned}\omega_A + \omega_B &= 1 \\ \beta_{A2} \omega_A + \beta_{B2} \omega_B &= 0.5\end{aligned}$$

Replace  $\omega_B$  with  $1 - \omega_A$  in the second equation and solve for  $\omega_A$ . It follows that  $\omega_A = 0.71$ ;  $\omega_B = 0.29$ ; and the exposure to  $F_1$  is 0.57.

#### Part c. (0.4 points)

Consider the Carhart 4-factor model, which is an extension of the Fama-French 3-factor model:

$$r_i = \alpha_i + \beta_i r_M + \gamma_i SMB + \delta_i HML + \kappa_i MOM + e_i$$

where  $r_i$  is the return of a stock  $i$ ,  $r_M$  is the market return,  $SMB$  is a factor that proxies for size,  $HML$  – for value, and  $MOM$  – for momentum. The variances of the four factors are respectively  $\sigma_M^2$ ,  $\sigma_{SMB}^2$ ,  $\sigma_{HML}^2$ , and  $\sigma_{MOM}^2$ . The variance of the idiosyncratic source of risk is  $\sigma_e^2$  for all stocks  $i$ . Assume that the idiosyncratic sources of risk are uncorrelated, and that the factors are uncorrelated as well.

Now consider three stocks ( $i=\{1, 2, 3\}$ ).

- i. Give an expression for the systematic risk of each of the three stocks.

$$\sigma_i^2 = \beta_i^2 \sigma_M^2 + \gamma_i^2 \sigma_{SMB}^2 + \delta_i^2 \sigma_{HML}^2 + \kappa_i^2 \sigma_{MOM}^2$$

- ii. Construct an equally weighted portfolio of the three stocks. What is its non-systematic risk component? Compare it to the non-systematic risk component of the individual stocks.

$$\sigma_P^2 = \frac{1}{3} \sigma_e^2$$

- iii. Construct a portfolio out of the three stocks that has exposure of 1 to the Size factor and an exposure of 0.5 to the Value factor. Provide the system of equations to be used to solve for the weights. You do not need to find the explicit solution for the weights.

The weights solve the following system of equations:

$$\begin{aligned} w_1 + w_2 + w_3 &= 1 \\ \gamma_1 w_1 + \gamma_2 w_2 + \gamma_3 w_3 &= 1 \\ \delta_1 w_1 + \delta_2 w_2 + \delta_3 w_3 &= 0.5 \end{aligned}$$

## Question 2. Portfolio Construction and Performance Measurement (1 point at maximum)

### Part a. (0.3 points)

Discuss the characteristics of indifference curves, and the theoretical value of these curves in the portfolio building process.

Indifference curves represent the trade-off between two variables. In portfolio building, the choice is between risk and return. The investor is indifferent between all possible portfolios lying on the indifference curve. However, indifference curves are contour maps, with all curves parallel to each other. The curve plotting in the most northwest position is the curve offering the greatest utility to the investor. However, this most desirable curve may not be attainable at the market place. The point of tangency between an indifference curve (representing what is desirable) and the capital allocation line (representing what is possible), is the optimum portfolio for that investor.

### Part b. (0.3 points)

Theoretically, the standard deviation of a portfolio consisting of two equities can be reduced to what level? Explain. Realistically, is it possible to reduce the standard deviation to this level? Explain.

Theoretically, if one could find two securities with perfectly negatively correlated returns (correlation coefficient = -1), one could solve for the weights of these securities that would produce the minimum variance portfolio of these two securities. The standard deviation of the resulting portfolio would be equal to zero. However, in reality, securities with perfect negative correlations do not exist.

**Part c. (0.4 points)**

You want to evaluate three mutual funds. The market return is 11% and the risk free rate is 3%.

Below is the data for three funds.

Fund	Average Return	Standard Deviation	Beta
<b>A</b>	13%	14%	1.1
<b>B</b>	11%	12%	1
<b>C</b>	8%	9%	0.8

- i. Provide the formulas for the Sharpe Ratio, Treynor measure, and Jensen's alpha.

Sharpe ratio = (return of the portfolio – risk free rate) / std. deviation

Treynor's measure = (return of the portfolio – risk free rate) / beta

Jensen's alpha = return of the portfolio – (risk free rate + beta \* (return of the market – risk free rate))

- ii. As an investor, would you prefer your fund to have higher or lower scores on these three measures? Why?

You would prefer a higher score. For the all three measures, a higher value indicates a higher fund performance given a measure of risk, which is desirable.

- iii. Calculate the Sharpe Ratio, Treynor measure, and Jensen's alpha. Based on your analysis, which fund performs best?

	Jensen's Alpha	SR	Treynor
<b>A</b>	1.20%	0.71	0.09
<b>B</b>	0.00%	0.67	0.08
<b>C</b>	-1.40%	0.56	0.06

- iv. Rank the funds from best performing to worst performing, based on your analysis.

Fund A > Fund B > Fund C

**Question 3. Fixed Income (1 point at maximum)****Part a. (0.3 points)**

Explain what the following terms mean: spot rate, short rate, and forward rate. Which of these is (are) observable today?

The n-period spot rate is the yield to maturity on a zero-coupon bond with a maturity of n periods. The short rate for period n is the one-period interest rate that will prevail in period n. The forward rate for period n is the short rate that would satisfy a "break-even condition" equating the total returns on two n-period investment strategies. The first strategy is an investment in an n-period zero-coupon bond. The second is an investment in an n-1 period zero-coupon bond "rolled over" into an investment in a one-period zero. Spot rates and forward rates are observable today, but because interest rates evolve with uncertainty, future short rates are not.

### Part b. (0.5 points)

Consider the data on the following three coupon bonds:

Bond	Maturity	Coupon	Yield	Face Value
A	2	0.07	0.02	100
B	3	0.06	0.03	100
C	4	0.04	0.04	100

- i. Compute the prices, duration and the modified duration of the three bonds.

For bond A:

$$P_A = 7/1.02 + 107/1.02^2 = 109.71$$

$$D_A = 1 \cdot 7/1.02/109.71 + 2 \cdot 107/1.02^2/109.71 = 1.94$$

$$D_A^* = 1.94/(1+0.02) = 1.90$$

You can easily follow the steps for the other two bonds

	Price	Duration	Modified Duration
Bond A	109.71	1.94	1.90
Bond B	108.49	2.84	2.76
Bond C	100.00	3.78	3.63

- ii. You have a portfolio consisting of a long position in 3 bonds of type A, 4 bonds of type B, and a short position in 1 bonds of type A. Calculate the duration and the modified duration of the portfolio.

$$\text{Value of the portfolio: } 3 \cdot P_A + 4 \cdot P_B - 1 \cdot P_A = 653.36$$

$$\text{Weights of the portfolio: } w_A = 3 \cdot P_A / 653.36 = 0.34. \text{ Same principle for the other 2.}$$

	Position	weights
Bond A	2 = 3-1	0.34
Bond B	4	0.66
Bond C	0	0.00

$$\text{Portfolio duration} = w_a \cdot D_a + w_b \cdot D_b + w_c \cdot D_c = 2.54$$

$$\text{Portfolio modified duration: } D^* = w_a \cdot D^*_a + w_b \cdot D^*_b + w_c \cdot D^*_c = 2.47$$

- iii. Using duration approximation, what is the change in the value of the portfolio if the yield curve shifts downwards by 100 basis points? And if it shifts downwards by 10 basis points? In which of the two cases the approximation will be more exact and why?

$$100\text{bp: } \Delta P/P = -2.54 \cdot -0.01 = 0.25$$

$$10\text{bp: } \Delta P/P = 0.02 \text{ (more exact as smaller yield change).}$$

#### Part c. (0.2 points)

Although the expectations of increases in future interest rates can result in an upward sloping yield curve; an upward sloping yield curve does not in and of itself imply the expectations of higher future interest rates. Explain.

The effects of possible liquidity premiums confound any simple attempt to extract expectation from the term structure. That is, the upward sloping yield curve may be due to expectations of interest rate increases, or due to the requirement of a liquidity premium, or both. The liquidity premium could more than offset expectations of decreased interest rates, and an upward sloping yield would result.

#### Question 4. Derivatives (1 point at maximum)

##### Part a. (0.2 points)

Discuss the relationship between option prices and a) volatility of the underlying stock, and b) the exercise price.

The greater the volatility of the underlying stock, the greater the option premium; the more volatile the stock, the more likely it is that the option will become more valuable (e. g., move from an out of the money to an in the money option, or become more in the money). For call options, the lower the exercise price, the more valuable the option, as the option owner can buy the stock at a lower price. For a put option, the lower the exercise price, the less valuable



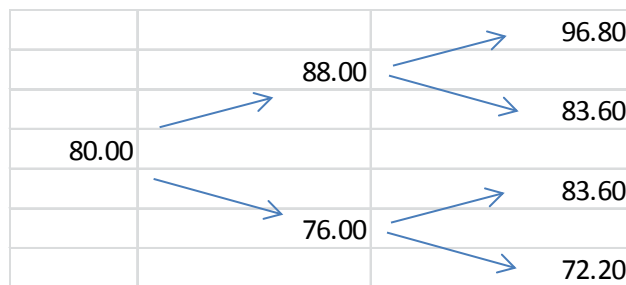
the option, as the owner of the option may be required to sell the stock at a lower than market price.

**Part b. (0.3 points)**

Consider a stock with a current price of 80 euro. Further, consider a binomial tree for the evolution of the price of the stock over the period of 1 year, assuming two steps ( $t=0$ ,  $t=1$ ,  $t=2$ ). At each nod, the price can go up by a factor of 1.1, or go down by a factor of 0.95. The **annual** risk free rate is 2%.

- i. Draw the binomial tree for ( $t=0$ ,  $t=1$ ,  $t=2$ )

The tree looks in the following way:



- ii. Calculate the risk-neutral probabilities of an upward movement and that of a downward movement. Do they differ at each nod of the tree and why?

The risk-neutral probability of an upward move is calculated as  $Q = \frac{S_0 * (1+r_f) - S_1^d}{S_1^u - S_1^d} = 0.4$ . Pay attention: the periodic risk free in this case is 1%. Consequently, the risk neutral probability of a downward movement is  $1-0.4=0.6$

The risk neutral probabilities are the same at each nod of the three, because they use the exact same discounting at each point in time.

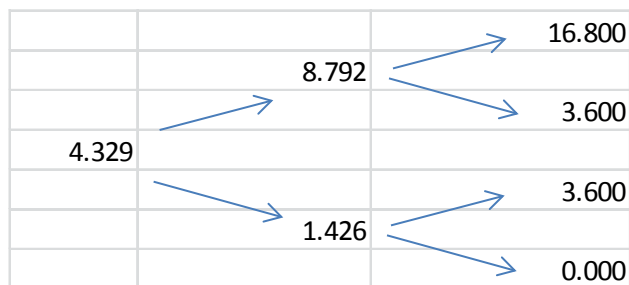
- iii. Suppose you consider buying an **at the money** call option that expires in 1 year.

X is 80. At time  $t=2$  the option payoffs at each end of the three are calculated as

$$C_i = \max(0, S_i - 110)$$

We then use the calculated risk-neutral probabilities to solve recursively:

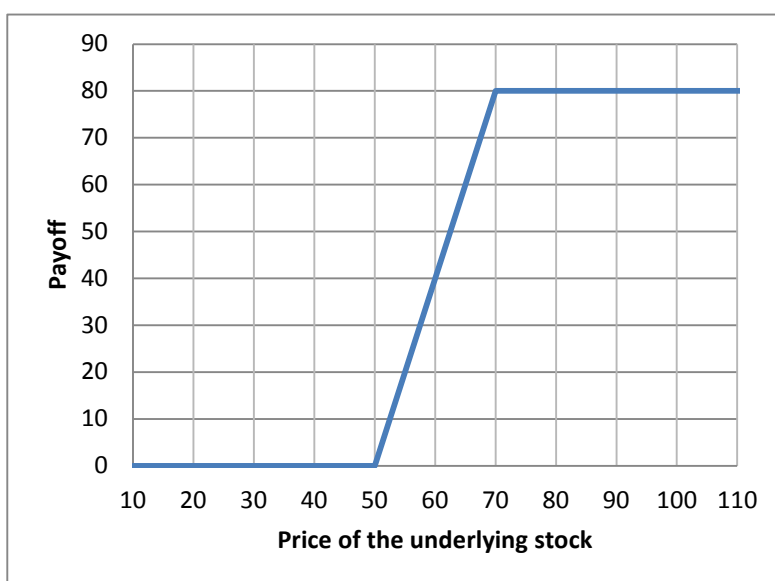
$$C_0 = Q \frac{C_1^u}{1+r_f} + (1-Q) \frac{C_1^d}{1+r_f}$$



Thus, the value of the call is 4.33 euro

**Part c. (0.5 points)**

The payoff of a collar strategy is given below.



Replicate the payoff using:

- i. A combination of call options  
Long 4 calls at 50, short 4 calls at 70
- ii. A combination of a cash account and put options  
Lend PV(80), long 4 puts at 50, short 4 puts at 70