vrije Universiteit amsterdam

Faculty of Economics and Business Administration

Exam:	Investments 3.4
Code:	E_BE3_INV
Examinator:	Dr. Teodor Dyakov
Co-reader:	Dr. Tanja Artiga Gonzalez
Date:	March 21, 2016
Time:	8.45
Duration:	2 hours and 45 minutes
Calculator allowed:	Yes
Graphical calculator allowed:	Yes
Number of questions:	30 multiple choice questions and 4 open-ended questions
Type of questions:	Open questions / multiple choice questions
Answer in:	English

Remarks: Be concise and complete in your answers (including calculations). Always explain the answers to the open questions, even if not explicitly called for. Use your time efficiently, using the maximum number of points per question as a guideline

Credit score:	You can receive between 0 and 9 points for the exam.
Grades:	The grades will be made public on: Wednesday, March 30 2016.
Inspection:	Friday, April 1 2016 at 09.00. Room number: to be announced on bb.
Number of pages:	19 (including front page)

Good luck!

PART 1 (MULTIPLE CHOICE QUESTIONS; 30 questions providing 5 points at maximum)

Read the questions and answers carefully and write down your answer on your answer sheet. Your final score is determined as (number of correct answers - 4)*5/26. Negative scores for this part of the exam are set to zero.

PART 2 (OPEN QUESTIONS; 4 questions providing 4 points at maximum)

1. When a distribution is negatively skewed, ____________.

A. standard deviation overestimates risk

B. standard deviation correctly estimates risk

C. standard deviation underestimates risk

D. the tails are fatter than in a normal distribution

E. the tails are skinnier than in a normal distribution

When a distribution is negatively skewed standard deviation underestimates risk.

2. A reward-to-volatility ratio is useful in:

A. measuring the standard deviation of returns.

B. understanding how returns increase relative to risk increases.

C. analyzing returns on variable rate bonds.

D. assessing the effects of inflation.

E. None of these is correct.

A reward-to-volatility ratio is useful in understanding how returns increase relative to risk increases.

3. When borrowing and lending at a risk-free rate are allowed, which Capital Allocation Line (CAL) should the investor choose to combine with the efficient frontier?

I) The one with the highest reward-to-variability ratio.

II) The one that will maximize his utility.

III) The one with the steepest slope.

IV) The one with the lowest slope.

A. I and III

B. I and IV

C. II and IV

D. I only

E. I, II, and III

The optimal CAL is the one that is tangent to the efficient frontier. This CAL offers the highest reward-to-variability ratio, which is the slope of the CAL. It will also allow the investor to reach his highest feasible level of utility.

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4. The standard deviation of a two-asset portfolio is a linear function of the assets' weights when

A. the assets have a correlation coefficient less than zero.

B. the assets have a correlation coefficient equal to zero.

C. the assets have a correlation coefficient greater than zero.

D. the assets have a correlation coefficient equal to one.

E. the assets have a correlation coefficient less than one.

When there is a perfect positive correlation (or a perfect negative correlation), the equation for the portfolio variance simplifies to a perfect square. The result is that the portfolio's standard deviation is linear relative to the assets' weights in the portfolio.

5. The index model has been estimated for stocks A and B with the following results:

$R_A = 0.01 + 0.8R_M + e_A$

$R_B = 0.02 + 1.1R_M + e_B$

$\sigma_M = 0.30$ $\sigma(e_A) = 0.20$ $\sigma(e_B) = 0.10$

The covariance between the returns on stocks A and B is ___________.

A. 0.0384

B. 0.0406

C. 0.1920

D. 0.0050

E. 0.0792

Cov(RA,RB) = bAbBs^{2}M = 0.8(1.1)(0.30)^2 = 0.0792.

6. Your opinion is that security A has an expected rate of return of 0.145. It has a beta of 1.5. The risk-free rate is 0.04 and the market expected rate of return is 0.11. According to the Capital Asset Pricing Model, this security is

A. underpriced.

B. overpriced by more than 2%.

C. fairly priced.

D. cannot be determined from data provided.

E. overpriced by less than 2%.

14.5% = 4% + 1.5(11% - 4%) = 14.5%; therefore, the security is fairly priced.

7. The CAPM applies to

A. portfolios of securities only.

B. individual securities only.

C. efficient portfolios of securities only.

D. efficient portfolios and efficient individual securities only.

E. all portfolios and individual securities.

The CAPM is an equilibrium model for all assets. Each asset's risk premium is a function of its beta coefficient and the risk premium on the market portfolio.

8. In the APT model, what is the nonsystematic standard deviation of an equally-weighted portfolio that has an average value of $\sigma(e_i)$ equal to 20% and 20 securities?

A. 12.5%

B. 625%

C. 4.47%

D. 3.54%

E. 14.59%

$$\sigma^2(e_p) = \frac{1}{n}\overline{\sigma}^2(e_i) = \frac{1}{20}(20)^2 = 20.0, \quad \sigma(e_p) = \sqrt{20.0} = 4.47\%$$

9. Google has a beta of 1.0. The annualized market return yesterday was 11%, and the risk-free rate is currently 5%. You observe that Google had an annualized return yesterday of 14%. Assuming that markets are efficient, this suggests that

A. bad news about Google was announced yesterday.

B. good news about Google was announced yesterday.

C. no news about Google was announced yesterday.

D. interest rates rose yesterday.

E. interest rates fell yesterday.

AR = 14% - (5% + 1.0 (6%)) = +3.0%. A positive abnormal return suggests that there was firm-specific good news.

10. Errors in information processing can lead investors to misestimate __________.

A. true probabilities of possible events and associated rates of return

B. only occurrences of possible events

C. only possible rates of return

D. the effect of accounting manipulation

E. fraud

Errors in information processing can lead investors to misestimate true probabilities of possible events and associated rates of return.

11. Tests of the CAPM that use regression techniques are subject to inaccuracies because

A. the statistical results used are almost always incorrect.

B. the slope coefficient of the regression equation is biased downward.

C. the slope coefficient of the regression equation is biased upward.

D. the intercept of the regression equation is biased downward.

E. the intercept of the regression equation is equal to the risk-free rate.

This would be a problem even if it were possible to use the returns on the true market portfolio in these regressions. It is due to the fact that the independent variable (the beta that is found in the first-pass regression and used as the independent variable in the second-pass regression) is measured with error.

The following data are available relating to the performance of Long Horn Stock Fund and the market portfolio:

	Long Horn	Market Portfolio
Average Return	19%	12%
Standard Deviation of Returns	35%	15%
Beta	1.5	1.0
Residual standard deviation	3.0%	0.0%

The risk-free return during the sample period was 6%.

12. What is the Treynor measure of performance evaluation for Long Horn Stock Fund?

A. 1.33%

B. 4.00%

C. 8.67%

D. 31.43%

E. 37.14%

(19% - 6%)/1.5 = 8.67%.

13. Calculate the information ratio for Long Horn Stock Fund.

A. 1.33

B. 4.00

C. 8.67

D. 31.43

E. 37.14

α_P = 19% - [6% + 1.5(12% - 6%)] = 4.00%, 4.00%/3.00% = 1.33.

14. ______ uses quantitative techniques and often automated trading systems to seek out many temporary misalignments among securities.

A. Covered interest arbitrage

B. Locational arbitrage

C. Triangular arbitrage

D. Statistical arbitrage

E. All arbitrage

Statistical arbitrage uses quantitative techniques and often automated trading systems to seek out many temporary misalignments among securities.

15. The yield to maturity on a bond is ________.

A. below the coupon rate when the bond sells at a discount, and equal to the coupon rate when the bond sells at a premium

B. the discount rate that will set the present value of the payments equal to the bond price

C. based on the assumption that any payments received are reinvested at the coupon rate

D. None of these are correct.

E. the discount rate that will set the present value of the payments equal to the bond price, and based on the assumption that any payments received are reinvested at the coupon rate.

The yield to maturity on a bond is the discount rate that will set the present value of the payments equal to the bond price.

16. Consider a 5-year bond with a 10% coupon that has a present yield to maturity of 8%. If interest rates remain constant, one year from now the price of this bond will be _______.

A. higher

B. lower

C. the same

D. cannot be determined

E. $1,000

This bond is a premium bond as interest rates have declined since the bond was issued. If interest rates remain constant, the price of a premium bond declines as the bond approaches maturity.

Par Value	$1,000
Time to Maturity	18 years
Coupon	9% (paid annually)
Current Price	$917.99
Yield to Maturity	10%

17. Given the bond described above, if interest were paid semi-annually (rather than annually), and the bond continued to be priced at $917.99, the resulting effective annual yield to maturity would be:

A. Less than 10%

B. More than 10%

C. 10%

D. Cannot be determined

E. None of these is correct.

FV = 1000, PV = -917.99, PMT = 45, n = 36, i = 4.995325 (semi-annual); $(1.4995325)^2$ - 1 = 10.24%. Another way to look at it: if coupon frequencies increase, then duration decreases and we discount over a short time period (on average). Hence, to keep the same price, the yield has to go up.

18. An inverted yield curve is one

A. with a hump in the middle.

B. constructed by using convertible bonds.

C. that is relatively flat.

D. that plots the inverse relationship between bond prices and bond yields.

E. that slopes downward.

An inverted yield curve occurs when short-term rates are higher than long-term rates.

The following is a list of prices for zero coupon bonds with different maturities and par value of $1,000.

Maturity (Years)	Price
1	$925.16
2	$862.57
3	$788.66
4	$711.00

19. What is the price of a 4-year maturity bond with a 10% coupon rate paid annually? (Par value = $1,000)

A. $742.09

B. $1,222.09

C. $1,035.66

D. $1,141.84

E. None of these is correct.

$(1000/711.00)^{1/4} - 1 = 8.9\%$; FV = 1000, PMT = 100, n = 4, i = 8.9, PV = $1,035.66

20. The duration of a 20-year zero-coupon bond is

A. equal to 20.

B. larger than 20.

C. smaller than 20.

D. equal to that of a 20-year 10% coupon bond.

E. None of these is correct.

Duration of a zero-coupon bond equals the bond's maturity.

21. Which of the following bonds has the longest duration?

A. A 12-year maturity, 0% coupon bond.

B. A 12-year maturity, 8% coupon bond.

C. A 4-year maturity, 8% coupon bond.

D. A 4-year maturity, 0% coupon bond.

E. Cannot tell from the information given.

The longer the maturity and the lower the coupon, the greater the duration.

22. One of the problems with immunization are

A. duration assumes that the yield curve is not flat.

B. duration assumes that if shifts in the yield curve occur, these shifts are parallel.

C. immunization is valid for multiple interest rate changes.

D. durations and horizon dates do not change by the same amounts with the passage of time.

E. duration assumes that students taking Investments 3.4 will all receive a grade of 10.

Duration assumes that if shifts in the yield curve occur, these shifts are parallel.

23. Speculators buying put options anticipate the value of the underlying asset will __________ and speculators selling call options anticipate the value of the underlying asset will _______.

A. increase; increase

B. decrease; increase

C. increase; decrease

D. decrease; decrease

E. cannot tell without further information

The buyer of the put option hopes the price will fall in order to exercise the option and sell the stock at a price higher than the market price. Likewise, the seller of the call option hopes the price will decrease so the option will expire worthless.

24. A collar with a net outlay of approximately zero is an options strategy that

A. combines a put and a call to lock in a price range for a security.

B. uses the gains from sale of a call to purchase a put.

C. uses the gains from sale of a put to purchase a call.

D. combines a put and a call to lock in a price range for a security and uses the gains from sale of a call to purchase a put.

E. combines a put and a call to lock in a price range for a security and uses the gains from sale of a put to purchase a call.

The collar brackets the value of a portfolio between two bounds.

25. If the hedge ratio for a stock call is 0.70, the hedge ratio for a put with the same expiration date and exercise price as the call would be _______.

A. 0.70

B. 0.30

C. -0.70

D. -0.30

E. -.17

Call hedge ratio = N(d1); Put hedge ratio = N(d1) - 1; 0.7 - 1.0 = -0.3.

26. The intrinsic value of an out-of-the-money call option is equal to

A. the call premium.

B. zero.

C. the stock price minus the exercise price.

D. the striking price.

E. None of these is correct.

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The fact that the owner of the option can buy the stock at a price greater than the market price gives the contract an intrinsic value of zero, and the holder will not exercise.

27. The Black-Scholes formula assumes that
I) the risk-free interest rate is constant over the life of the option.
II) the stock price volatility is constant over the life of the option.
III) the expected rate of return on the stock is constant over the life of the option.
IV) there will be no sudden extreme jumps in stock prices.
A. I and II
B. I and III
C. II and II
D. I, II and IV
E. I, II, III, and IV

The risk-free rate and stock price volatility are assumed to be constant but the option value does not depend on the expected rate of return on the stock. The model also assumes that stock prices will not jump markedly.

28. Which of the following is **true** about profits from futures contracts?
A. The person with the long position gets to decide whether to exercise the futures contract and will only do so if there is a profit to be made.
B. It is possible for both the holder of the long position and the holder of the short position to earn a profit.
C. The clearinghouse makes most of the profit.
D. The amount that the holder of the long position gains must equal the amount that the holder of the short position loses.
E. Holders of short positions can recognize profits by making delivery early.

The net profit on the contract is zero - it is a zero-sum game.

29. Suppose that the risk-free rates in the United States and in the Canada are 5% and 3%, respectively. The spot exchange rate between the dollar and the Canadian dollar (C$) is $0.80/C$. What should the futures price of the C$ for a one-year contract be to prevent arbitrage opportunities, ignoring transactions costs.
A. $1.00/ C$
B. $0.82/ C$
C. $0.88/ C$
D. $0.78/ C$
E. $1.22/ C$

$0.80(1.05/1.03) = $0.82/ C$.

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30. Credit risk in the swap market

A. is extensive.

<u>B.</u> is limited to the difference between the values of the fixed rate and floating rate obligations.

C. is equal to the total value of the payments that the floating rate payer was obligated to make.

D. is extensive and is equal to the total value of the payments that the floating rate payer was obligated to make.

E. None of these is correct.

Swaps obligate two counterparties to exchange cash flows at one or more future dates. Swaps allow firms to restructure balance sheets, and the firm is obligated only for the difference between the fixed and floating rates.

PART 2 (OPEN QUESTIONS; 4 questions providing 4 points at maximum)

Read the questions and answers carefully and write down your answer on your answer sheet.

Question 1. Equilibrium Pricing Models (1 point at maximum)

Part a. (0.3 points)

You have a sample of n stocks with returns $r_{\{i,t\}}$ and the periodic risk free rate is given by $r_{\{f,t\}}$. You want to test some of the implications of the CAPM using the following regression:

$$\widehat{r_i - r_f} = \gamma_0 + \gamma_1 * \beta_i + \gamma_2 * \sigma^2(\epsilon_i)$$

where $i = 1, \ldots, n$ and $\widehat{r_i - r_f}$ is the average excess return on stock i, β_i is stock i's beta coefficient, and $\sigma^2(\epsilon_i)$ is the variance of stock i's non-systematic component.

i. The above equation is known as a second-pass regression equation. Give the equation of the first-pass regression. Is the first pass-regression run per stock (one time-series regression per stock), or is it run over all stocks (one panel regression using the information for all stocks? Which of the output from the first-pass regression is used for the second-pass regression given above?

   We have to run a time series regression for each stock (thus one regression for each of the n stocks):

   $$r_{\{i,t\}} = \alpha_t + \beta_i\left(r_{\{M,t\}} - r_{\{f,t\}}\right) + \epsilon_i$$

   The estimated betas and standard deviation of the residual enter the second-pass regression.

ii. State the hypotheses concerning the gamma coefficients if the CAPM is valid. Explain.

$$\gamma_0 = 0;\ \gamma_1 = \overline{r_{\{M,t\}} - r_{\{f,t\}}};\ \gamma_2 = 0$$

Part b. (0.3 points)

Discuss the similarities and the differences between the CAPM and the APT with regard to the following factors: capital market equilibrium, assumptions about risk aversion, risk-return dominance, and the number of investors required to restore equilibrium.

Both the CAPM and the APT are market equilibrium models, which examine the factors that affect securities' prices. In equilibrium, there are no overpriced or underpriced securities. In both models, mispriced securities can be identified and purchased or sold as appropriate to earn excess profits.

The CAPM is based on the idea that there are large numbers of investors who are focused on risk-return dominance. Under the CAPM, when a mispricing occurs, many individual investors make small changes in their portfolios, guided by their degrees of risk aversion. The aggregate effect of their actions brings the market back into equilibrium. Under the APT, each investor wants an infinite arbitrage position in the mispriced asset. Therefore, it would not take many investors to identify the arbitrage opportunity and act to bring the market back to equilibrium.

Part c. (0.4 points)

Consider the multifactor APT. There are two independent economic factors, F1 and F2.The risk-free rate of return is 3%. The following information is available about two well-diversified portfolios A and B:

Portfolio	beta on F1	beta on F2	expected return	variance
A	1	0.5	7%	13%
B	0	1	6%	12%

i. Assuming no arbitrage opportunities exist, calculate the risk premia on the two factor Portfolios

The risk premia of the two factors should solve the following system of equations

7%=3% + 1*RP1 + 0.5*RP2
6%=3% + 0*RP1 + 1*RP2

It follows the RP2=3% and RP1=2.5%

ii. Give an expression for the variance of a well-diversified portfolio along the lines of the multifactor APT. Calculate the variance of the two factors, given the data above.

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Because A and B are well-diversified:

$$\beta_{A1}^2\sigma_{F1}^2 + \beta_{A2}^2\sigma_{F2}^2 = \sigma_A^2$$
$$\beta_{B1}^2\sigma_{F1}^2 + \beta_{B2}^2\sigma_{F2}^2 = \sigma_B^2$$

Solve first σ_B^2, since $\beta_{B1}^2 = 0$. Thus, $\sigma_B^2 = 12\%$. Plug it back in the first equation and solve for σ_A^2. The answer is $\sigma_A^2 = 10\%$

iii. Construct a portfolio of A and B that has exposure of 0.7 to F_2. What are the weights of the two portfolios A and B? What is the exposure to F_1 of the newly created portfolio?

The weights of A and B should satisfy the following system of equations:

$$\omega_A + \omega_B = 1$$
$$\beta_{A1}\omega_A + \beta_{B1}\omega_B = 0.7$$

Replace ω_B with $1 - \omega_A$ in the second equation and solve for ω_A. It follows that $\omega_A = 0.6$; $\omega_B = 0.4$; and the exposure to F_1 is 0.7.

Question 2. Portfolio Construction and Performance Measurement (1 point at maximum)

Part a. (0.3 points)

You solve a portfolio allocation problem, including a number of risky assets and a risk-free rate. You've found the optimal mix of risky assets. Further, you've showed the optimal proportion of the portfolio of risky assets in the complete portfolio is given by the equation:

$$w^* = \frac{E[r_E] - r_f}{A\sigma_E^2}$$

For each of the variables on the right side of the equation, discuss the impact of the variable's effect on w* and why the nature of the relationship makes sense intuitively. Assume the investor is risk averse.

The optimal proportion in the risky portfolio (the one containing the optimal mix of risky assets) is the one that maximizes the investor's utility. Utility is positively related to the risk premium [$E(r_e)$-r_f]. This makes sense because the more expected return an investor gets, the happier she is. The variable "A" represents the degree of risk aversion. As risk aversion increases, "A" increases. This causes w* to decrease because we are dividing by a higher number. It makes sense that a more risk-averse investor would hold a smaller proportion of his complete portfolio in the risky asset and a higher proportion in the risk-free asset. Finally, the standard deviation of the risky portfolio is inversely related to w*. As the risky portfolio's risk increases, we are again dividing by a larger number, making w* smaller. This corresponds with the risk-averse investor's dislike of risk as measured by standard deviation.

Part b. (0.3 points)

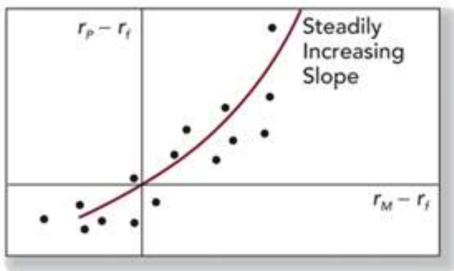
You are evaluating the market timing ability of a portfolio manager. In order to do so, you use the following regression equation:

$$r_P - r_f = a + b\left(r_M - r_f\right) + c\left(r_M - r_f\right)^2 + e_P$$

where r_p is the return of the portfolio, r_f is the risk-free rate, r_m is the market return and e_p is the error term.

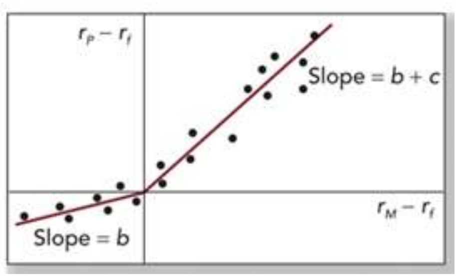
i. How would you test for the market timing ability of the manager, using the above regression equation? Provide a graphical interpretation.

Test for c>0. Finding c>0 would indicate market timing ability.



ii. How would you define an ideal situation of perfect foresight, (using a graphical interpretation). Hint: think of a derivative instrument.

Perfect foresight is equivalent to holding a call option on the index portfolio.



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Part c. (0.4 points)

Consider two perfectly negatively correlated risky securities X and Z. X has an expected rate of return of 10% and a standard deviation of 15%, and Z has an expected return of 5% and a standard deviation of 7%. You want to construct a portfolio out of the two securities that has as small variance as possible.

i. What is the formula for the variance of the portfolio?

$$\sigma_P^2 = \left(w_E\sigma_E - w_D\sigma_D\right)^2$$

ii. What is the variance of the portfolio?

It is 0, since the two assets are perfectly negatively correlated.

iii. What proportions of X and Z should you hold in the portfolio?

w1=7/(15+7)=0.32; w2=1-w1=0.68

iv. What is the expected return of the portfolio?

w1*ER1 + w2*ER2=0.66

Question 3. Fixed Income (1 point at maximum)

Part a. (0.3 points)

Consider the following forward rates: f_1 = 1%, f_2 = 2%, and f_3 = 3%.

i. Express the prices of zero-coupon bonds with maturities of 1, 2, and 3 years using the forward rates. The face value of each bond is 1000 euro. Solve for them.

$P_1 = 1000/(1+f_1) = 990.10$, $P_2 = 1000/((1+f_1)*(1+f_2)) = 970.69$, $P_3 = 1000/((1+f_1)*(1+f_2)*(1+f_3)) = 942.41$.

ii. Explain how you can obtain the zero-yield curve, using the forward rates. That is, provide the formulas for computing the yields of zero-coupon bonds with maturities 1, 2, 3.

$z_1 = f_1$

$(1+z_2)^2 = (1+f_1)*(1+f_2)$

$(1+z_3)^3 = (1+f_1)*(1+f_2)*(1+f_3)$

iii. Calculate the yield curve

Solving this system above yields: $z_1 = 0.01$, $z_2 = 0.015$, $z_3 = 0.020$.

iv. Comment on the shape of the derived yield curve. Based on the expectations and liquidity preference theories, do you expect an increase in interest rates?

The yield curve is upward sloping. According to the expectations theory, an upward sloping curve indicates that investors anticipate an increase in interest rates. According to liquidity preference theory, an upward sloping curve does not necessarily imply an anticipation for an increase in interest rates.

Part b. (0.4 points)

Continue using the data provided in Part a. You are now offered to enter in the following agreement: lend 20000 euro to a financial company at the start of year 3 at 5% interest rate for a period of 1 year.

i. What is the value of the agreement today? How do you obtain the value?

$-20000/(1+z_2)^2 + 20000*(1+0.05)/(1+z_3)^2 = 376.97.$

ii. What are the cash-flows of the agreement (now (beginning of year 1), at the end of year 1, 2 and 3)? How can you hedge this agreement today? (Hint: use zero-coupon bonds). Give the portfolio for the hedging strategy and its cash-flows.

Cash-flows of the agreement:

now	376.97
1	0
2	-20000
3	20800

Hedging the agreement today:

now	Short 20 2-year zero bonds, Long 21 3-year zero bonds, value of the position: 20*970.69 - 21*942.41= - 376.97
1	0
2	-20*1000

.

3 21*1000

Note: If you did not solve correctly Part a. but solved correctly for Part b. using the incorrectly obtained values in Part a., you still receive full credit for part Part b.

Part c. (0.3 points)

What does credit risk mean in the context of bond pricing? Which agencies assess the credit risk of bond issuers? Explain briefly the rating scheme employed by those agencies.

A credit risk is the risk of default on a debt that may arise from a borrower failing to make required payments. Credit risk is measured by credit rating agencies, most important of which are Moody's, Standard & Poor's, and Fitch. Each credit rating agencies assigns letter grades to the bonds of corporations / governments / municipalities to reflect their assessment of the safety of the entity. The top rating is AAA or Aaa. Those rated BBB (Baa) or above by the credit agencies are considered investment grade bonds, whereas lower-rated bonds are classified as speculative grade or junk bonds.

Question 4. Derivatives (1 point at maximum)

Part a. (0.3 points)

State the put-call parity. Why is it called in such a way? Derive the put-call parity, using two options strategies that provide the same payoffs.

The put-call parity represents the proper relationship between put and call prices. More specifically,

$$\underbrace{c}_{\text{call price}} + \underbrace{\frac{X}{\left(1+r_f\right)^T}}_{\text{purchase price of the ZC bond}} = \underbrace{S_0}_{\text{purchase price of the stock}} + \underbrace{p}_{\text{put price}}$$

To derive the parity, consider two strategies:

- A protective put
- Buying a call option and a zero-coupon bond with a face value that equals the strike price of the call, and the same maturity

Now examine the pay-offs:

Protective put	$S_T \leq X$	$X < S_T$
Stock	S_T	S_T
+ Put	$X - S_T$	0
= Total	X	S_T

Call + ZC bond	$S_T \leq X$	$X < S_T$
Call	0	$S_T - X$
+ ZC riskless bond	X	X
= Total	X	S_T

Because the two portfolios provide the same return, they must cost the same. Thus, we reach the put-call parity.

Part b. (0.4 points)

You hold a portfolio of 56 million euro with a beta of 0.75 on the AEX. You expect that the AEX will drop by 2.5% over the next one year. You decide to hedge the AEX exposure of your portfolio with index futures with expiration of one year. The contract multiplier is 200 and the current value of the AEX is 420.

i. Do you buy or sell index futures in order to hedge your position?

You sell index futures

ii. What is the projected loss on your portfolio (in euro), if you do not hedge?

Beta*projected loss (in %)*portfolio value=0.75*0.025*56*10^6=1.05 mil

iii. How much would each index future contract change in value for the projected 5% drop in the AEX?

AEX change*AEX starting level*Contract multiplier=0.025*420*200=2100

iv. What is the hedge ratio, i.e. how many future contracts do you need for a perfect hedge?

Projected loss in euro / change in the value of the futures = 1.05*10^6/2100=500 contracts

Part c. (0.3 points)

Consider a stock with a current price of 120 euro. Further, consider a binomial tree for the evolution of the price of the stock over the period of 1 year, assuming two steps (t=0, t=1, t=2). At each nod, the price can go up by a factor of 1.1, or go down by a factor of 0.95. The **annual** risk free rate is 1%.

i. Draw the binomial tree for (t=0, t=1, t=2)

The tree looks in the following way:

		145.20
	132.00	
		125.40
120.00		
		125.40
	114.00	
		108.30

ii. Calculate the risk-neutral probabilities of an upward movement and that of a downward movement. Do they differ at each nod of the tree and why?

The risk-neutral probability of an upward move is calculated as $Q = \frac{S_0*(1+r_f)-S_1^d}{S_1^u-S_1^d} = 0.36(6)$. Pay attention: the periodic risk free in this case is 0.5%. Consequently, the risk neutral probability of a downward movement is 1-0.37=0.63

The risk neutral probabilities are the same at each nod of the three, because they use the exact same discounting at each point in time.

iii. Suppose you consider buying plain vanilla out of the money call options that expire in 1 year. Compute the price of such an option with a strike price of 110 euro.

X is 110. At time t=2 the option payoffs at each end of the three are calculated as

$$C_i = \max(0, S_i - 110)$$

We then use the calculated risk-neutral probabilities to solve recursively:

$$C_0 = Q\frac{C_1^u}{1+rf} + (1-Q)\frac{C_1^d}{1+rf}$$

		35.200
	22.547	
		15.400
11.767		
		15.400
	5.619	
		0.000

Thus, the value of the call is 11.77 euro