vrije Universiteit amsterdam

Faculty of Economics and Business Administration

Exam:	Investments 3.4
Code:	E_BE3_INV
Examinator:	Dr. Teodor Dyakov
Co-reader:	Dr. Anne Opschoor
Date:	May 18, 2015
Time:	8.30
Duration:	2 hours and 45 minutes
Calculator allowed:	Yes
Graphical calculator allowed:	Yes
Number of questions:	20 multiple choice and 4 open-ended
Type of questions:	Open/ multiple choice
Answer in:	English

Remarks: Be concise and complete in your answers (including calculations). Always explain your answers, even if not explicitly called for. Use your time efficiently, using the maximum number of points per question as a guideline

Credit score:	The maximum possible scores for each part and question are indicated. In total, you can earn 100 points. Your final exam grade is determined by dividing the number of points by 10.
Grades:	The grades will be made public on: June 1, 2015
Inspection:	Tuesday, June 2, 2015 at 13.00. Room – to be announced on blackboard.
Number of pages:	17 (including front page)

Good luck!

PART 1 (MULTIPLE CHOICE; 40 points at maximum)

Read the questions and answers carefully and write down your answer on your answer sheet. Your final score is determined as (# correct answers - 2) * 40/18. Negative scores for this part of the exam are set to zero.

1. Which of the following statements is (are) true regarding the variance of a portfolio of two risky securities?
A. The higher the coefficient of correlation between securities, the greater the reduction in the portfolio variance.
B. There is a linear relationship between the securities' coefficient of correlation and the portfolio variance.
C. The degree to which the portfolio variance is reduced depends on the degree of correlation between securities.
D. A and B.
E. A and C.

The lower the correlation between the returns of the securities, the more portfolio risk is reduced.

2. An investor who wishes to form a portfolio that lies to the right of the optimal risky portfolio on the Capital Allocation Line must:
A. lend some of her money at the risk-free rate and invest the remainder in the optimal risky portfolio.
B. borrow some money at the risk-free rate and invest in the optimal risky portfolio.
C. invest only in risky securities.
D. such a portfolio cannot be formed.
E. B and C

The only way that an investor can create portfolios to the right of the Capital Allocation Line is to create a borrowing portfolio (buy stocks on margin). In this case, the investor will not hold any of the risk-free security, but will hold only risky securities.

3. Suppose you held a well-diversified portfolio with a very large number of securities, and that the single index model holds. If the σ of your portfolio was 0.20 and σ_M was 0.16, the β of the portfolio would be approximately _____.
A. 0.64
B. 0.80
C. 1.25
D. 1.56
E. none of the above

$s^2p / s^2m = b^2$; $(0.2)^2/(0.16)^2 = 1.56$; $b = 1.25$.

4. An important difference between CAPM and APT is
A. CAPM depends on risk-return dominance; APT depends on a no arbitrage condition.
B. CAPM assumes many small changes are required to bring the market back to equilibrium; APT assumes a few large changes are required to bring the market back to equilibrium.
C. implications for prices derived from CAPM arguments are stronger than prices derived from APT arguments.
D. all of the above are true.
E. both A and B are true.

Under the risk-return dominance argument of CAPM, when an equilibrium price is violated many investors will make small portfolio changes, depending on their risk tolerance, until equilibrium is restored. Under the no-arbitrage argument of APT, each investor will take as large a position as possible so only a few investors must act to restore equilibrium. Implications derived from APT are much stronger than those derived from CAPM, making C an incorrect statement.

5. The risk-free rate is 7 percent. The expected market rate of return is 15 percent. If you expect a stock with a beta of 1.3 to offer a rate of return of 12 percent, you should
A. buy the stock because it is overpriced.
B. sell short the stock because it is overpriced.
C. sell the stock short because it is underpriced.
D. buy the stock because it is underpriced.
E. none of the above, as the stock is fairly priced.

12% < 7% + 1.3(15% - 7%) = 17.40%; therefore, stock is overpriced and should be shorted.

6. A finding that _____ would provide evidence against the semistrong form of the efficient market theory.
A. low P/E stocks tend to have positive abnormal returns
B. trend analysis is worthless in determining stock prices
C. one can consistently outperform the market by adopting a contrarian strategy based on price reversals
D. A and B
E. A and C

Both A and C are inconsistent with the semistrong form of the EMH.

7. Consider the regression equation:

$r_i - r_f = g_0 + g_1 b_i + g_2 s^2(e_i) + e_{it}$

where:

$r_i - r_t$ = the average difference between the monthly return on stock **i** and the monthly risk-free rate

b_i = the beta of stock **i**

$s^2(e_i)$ = a measure of the nonsystematic variance of the stock **i**

If you estimated this regression equation and the CAPM was valid, you would expect the estimated coefficient, g_1 to be

A. 0

B. 1

C. equal to the risk-free rate of return.

D. equal to the average difference between the monthly return on the market .portfolio and the monthly risk-free rate.

E. equal to the average monthly return on the market portfolio.

The variable measured by the coefficient g1 in this model is the market risk premium.

8. Each of two stocks, A and B, are expected to pay a dividend of $5 in the upcoming year. The expected growth rate of dividends is 10% for both stocks. You require a rate of return of 11% on stock A and a return of 20% on stock B. The intrinsic value of stock A _____.

A. will be greater than the intrinsic value of stock B

B. will be the same as the intrinsic value of stock B

C. will be less than the intrinsic value of stock B

D. cannot be calculated without knowing the market rate of return.

E. none of the above is true.

PV0 = D1/(k-g); given that dividends are equal, the stock with the larger required return will have the lower value.

9. Midwest Airline is expected to pay a dividend of $7 in the coming year. Dividends are expected to grow at the rate of 15% per year. The risk-free rate of return is 6% and the expected return on the market portfolio is 14%. The stock of Midwest Airline has a beta of 3.00. The return you should require on the stock is _____.

A. 10%

B. 18%

C. 30%

D. 42%

E. none of the above

6% + 3(14% - 6%) = 30%.

10. Suppose two portfolios have the same average return, the same standard deviation of returns, but portfolio A has a higher beta than portfolio B. According to the Sharpe measure, the performance of portfolio A _____.

A. is better than the performance of portfolio B

B. is the same as the performance of portfolio B

C. is poorer than the performance of portfolio B

D. cannot be measured as there is no data on the alpha of the portfolio

E. none of the above is true.

The Sharpe index is a measure of average portfolio returns (in excess of the risk free return) per unit of total risk (as measured by standard deviation).

11. A Treasury bond due in one year has a yield of 5.7%; a Treasury bond due in 5 years has a yield of 6.2%. A bond issued by Ford Motor Company due in 5 years has a yield of 7.5%; a bond issued by Shell Oil due in one year has a yield of 6.5%. The default risk premiums on the bonds issued by Shell and Ford, respectively, are

A. 1.0% and 1.2%

B. 0.7% and 1.5%

C. 1.2% and 1.0%

D. 0.8% and 1.3%

E. none of the above

Shell: 6.5% - 5.7% = .8%; Ford: 7.5% - 6.2% = 1.3%.

12. You have just purchased a 10-year zero-coupon bond with a yield to maturity of 10% and a par value of $1,000. What would your rate of return at the end of the year be if you sell the bond? Assume the yield to maturity on the bond is 11% at the time you sell.

A. 10.00%

B. 20.42%

C. 13.8%

D. 1.4%

E. none of the above

$1,000/(1.10)10 = $385.54; $1,000/(1.11)9 = $390.92; ($390.92 - $385.54)/$385.54 = 1.4%.

13. Suppose that all investors expect that interest rates for the 4 years will be as follows:

Year	Forward Interest Rate
0	(today)5%
1	7%
2	9%
3	10%

What is the price of 3-year zero coupon bond with a par value of $1,000?

A. $863.83

B. $816.58

C. $772.18

D. $765.55

E. none of the above

1000/(1.05*1.07*1.09)

14. Holding other factors constant, the interest-rate risk of a coupon bond is higher when the bond's:

A. term-to-maturity is lower.

B. coupon rate is lower.

C. yield to maturity is higher.

D. A and C

E. none of the above.

The longer the maturity, the greater the interest-rate risk. The lower the coupon rate, the greater the interest-rate risk. The lower the yield to maturity, the greater the interest-rate risk. These concepts are reflected in the duration rules; duration is a measure of bond price sensitivity to interest rate changes (interest-rate risk).

15. Some of the problems with immunization are

A. duration assumes that the yield curve is flat.

B. duration assumes that if shifts in the yield curve occur, these shifts are parallel.

C. immunization is valid for one interest rate change only.

D. durations and horizon dates change by the same amounts with the passage of time.

E. A, B, and C.

Durations and horizon dates change with the passage of time, but not by the same amounts.

16. A covered call position is
A. the simultaneous purchase of the call and the underlying asset.
B. the purchase of a share of stock with a simultaneous sale of a put on that stock.
C. the short sale of a share of stock with a simultaneous sale of a call on that stock.
D. the purchase of a share of stock with a simultaneous sale of a call on that stock.
E. the simultaneous purchase of a call and sale of a put on the same stock.

Writing a covered call is a very safe strategy, as the writer owns the underlying stock. The only risk to the writer is that the stock will be called away, thus limiting the upside potential.

17. The Black-Scholes formula assumes that
I) the risk-free interest rate is constant over the life of the option.
II) the stock price volatility is constant over the life of the option.
III) the expected rate of return on the stock is constant over the life of the option.
IV) there will be no sudden extreme jumps in stock prices.
A. I and II
B. I and III
C. II and II
D. I, II and IV
E. I, II, III, and IV

The risk-free rate and stock price volatility are assumed to be constant but the option value does not depend on the expected rate of return on the stock. The model also assumes that stock prices will not jump markedly.

18. Other things equal, the price of a stock call option is negatively correlated with the following factors
A. the stock price.
B. the time to expiration.
C. the stock volatility.
D. the exercise price.
E. A, B, and C.

The exercise price is negatively correlated with the call option price.

19. If interest rate parity holds, then
A. covered interest arbitrage opportunities will exist
B. covered interest arbitrage opportunities will not exist
C. the average grade on Investments 3.4 will be above 8.5
D. arbitrageurs will be able to make risk-free profits
E. B and D

If interest rate parity holds covered interest arbitrage opportunities will not exist

20. Suppose that the risk-free rates in the United States and in the United Kingdom are 4% and 6%, respectively. The spot exchange rate between the dollar and the pound is $1.60/BP. What should the futures price of the pound for a one-year contract be to prevent arbitrage opportunities, ignoring transactions costs.

A. $1.60/BP
B. $1.70/BP
C. $1.66/BP
D. $1.63/BP
E. $1.57/BP

$1.60(1.04/1.06) = $1.57/BP.

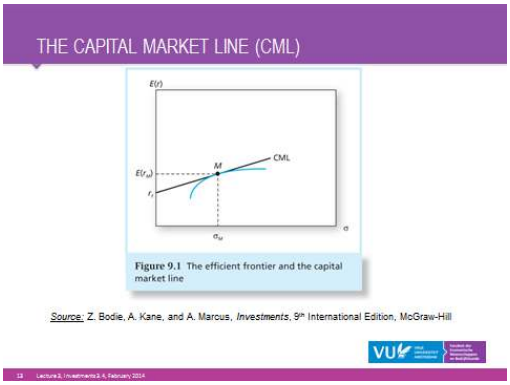
PART 2 (OPEN QUESTIONS; 60 points at maximum)

Question 1: Equilibrium Pricing Models (15 points)

Part a. (4 points)

What are the equilibrium implications of the CAPM? When answering this question focus on the following:

i. Give a graphical illustration of the Capital Market Line and explain it.
ii. State the Mutual Fund Theorem
iii. Give an expression for the risk premium of the market portfolio
iv. Given an expression for the risk premium of individual assets (the expected return-beta relationship)



All investors choose to hold the same (market) portfolio, which is the tangency portfolio to the efficient frontier: the one with the best attainable CAL. The allocation between the market portfolio and the risk free rate depends on individual investor's risk aversion. The risk premium of the market portfolio is given by:

$$E[r_m] - r_f = \bar{A}\sigma_m^2$$

where $E[r_m]$ is the expected market return, r_f is the risk-free rate, $\bar{A}$ is the degree of risk aversion of the representative investor, and σ_m^2 is the variance of the market portfolio. The risk premium of individual assets is given by:

$$E[r_i] = r_f + \beta_{i(E[r_m]-r_f)}$$

where $\beta_i = cov(r_m, r_i)/\sigma_m^2$

Part a. (4 points)

Consider the Fama-French 3-factor model:

$$r_i = \alpha_i + \beta_i * r_m + \gamma_i * SMB + \delta_i * HML + \epsilon_i$$

where r_i is the return of a stock i, r_m is the market return, SMB is a factor that proxies for size, and HML – for value. The variances of the three factors are respectively σ_m^2 , σ_{smb}^2 , and σ_{hml}^2. The variance of the idiosyncratic source of risk is σ_ϵ^2 and it is the same for all stocks i.
Assume that the idiosyncratic sources of risk are uncorrelated, and that the three factors are uncorrelated as well.

i. Give an expression for the systematic risk of stock i in terms of the variances of the three factors.

$$\sigma_i^2 = \beta_i^2 * \sigma_m^2 + \gamma_i^2 * \sigma_{smb}^2 + \delta_i^2 * \sigma_{hml}^2$$

ii. Construct an equally weighted portfolio of 10 stocks. What is its non-systematic risk component? Compare it to the non-systematic risk component of the individual stocks.

$$\sigma_P^2 = 0.1 * \sigma_\epsilon^2$$

iii. Consider three stocks, i={1,2,3}. Construct a portfolio out of the three stocks that has zero exposure to the size factor, and an exposure of 1.5 to the value factor. Provide the system of equations to be used to solve for the weights. You do not need to find the explicit solution for the weights.

$$w_1 + w_2 + w_3 = 1$$
$$\gamma_1 * w_1 + \gamma_2 * w_2 + \gamma_3 * w_3 = 0$$
$$\delta_1 * w_1 + \delta_2 * w_2 + \delta_3 * w_3 = 1.5$$

Part c. (4 points)

Assume that the CAPM holds. The risk-free rate is 2% and the market return is 8%. Consider the following information for two stocks A and B. Beta refers to the market beta, P/E – to price-earnings ratio, and b denotes the retention ratio:

Stocks	beta	P/E	b
A	1.5	14.29	0.7
B	0.9	13.51	0.5

i. Compute the return on equity (ROE) of the two stocks, assuming a constant growth model with endogenous earnings growth (round at the third decimal).

k=rf+beta*(rm-rf)
ROE=(P/E*k +1 –b)/(b*P/E)

	k	ROE
A	0.110	0.127
B	0.074	0.074

ii. Estimate the growth rate for both firms (round at the third decimal).

g=ROE*b

	g
A	0.089
B	0.037

iii. What would the effect on the P/E ratio be if you change the retention ratio of stock B, keeping ROE constant? Why?

It will not change since $ROE_B = k_B$.

Part d. (3 points)

Consider the multifactor APT with two factors. The risk premiums on the factor 1 and factor 2 portfolios are 5% and 6%, respectively. Stock A has a beta of 1.2 on factor 1, and a beta of 0.7 on factor 2. The expected return on stock A is 17%.

If no arbitrage opportunities exist, what should the risk-free rate of return be?

17% = x% + 1.2(5%) + 0.7(6%); x = 6.8%.

Question 2: Portfolio Construction and Performance Measurement (15 points)

Part a. (6 points)

Consider a portfolio P that consists of a risky asset A and a risk-free asset with weights ω and (1-ω) respectively. Denote by r_A the return of the risky asset, and by r_f – the risk-free return. The standard deviation of the risky asset is σ_A.

i. Give an analytic expression of the return of the portfolio (r_P) and its variance (σ_P^2). What is the expected return of the portfolio $E[r_P]$?

$$r_p = wr_A + (1-w)r_f$$
$$\sigma_p^2 = w^2\sigma_A^2$$
$$E[r_p] = wE[r_A] + (1-w)r_f$$

ii. Consider an investor who has the following utility function:

$$U = E[r_P] - \frac{1}{2}A\sigma_P^2$$

where A is the level of risk aversion of the investor. Obtain analytically the optimal weights in the risky asset ω if the investor solves the following problem:

$$max_w U = E[r_P] - \frac{1}{2}A\sigma_P^2$$
$$U = wE[r_A] + (1-w)r_f - \frac{1}{2}Aw^2\sigma_A^2$$

Setting its first derivative with respect to w to zero:

$$U' = E[r_A] - r_f - Aw\sigma_A^2 = 0$$

We obtain for the optimal weight:

$$w^* = \frac{E[r_A] - r_f}{A\sigma_A^2}$$

iii. Which value of A makes the investor indifferent between investing in the risky portfolio P and the risk-free asset? Give an analytic expression.

$$U(r_p) = U(r_f)$$
$$E[r_P] - \frac{1}{2}A\sigma_P^2 = r_f$$
$$A = \frac{2(E[r_P] - r_f)}{\sigma_P^2}$$

Part b. (5 points)

You want to evaluate three mutual funds, based on the information below. The market return is 8% and the risk free rate is 2%.

Fund	Average Return	Standard Deviation	Beta
A	9	9	0.7
B	12	16	1.3
C	14	20	1.8

i. Provide the formulas for the Sharpe Ratio, Treynor measure, and Jensen's alpha.

Sharpe ratio = (return of the portfolio – risk free rate) / std. deviation
Treynor's measure = (return of the portfolio – risk free rate) / beta
Jensen's alpha = return of the portfolio – (risk free rate + beta * (return of the market – risk free rate))

ii. As an investor, would you prefer your fund to have higher or lower scores on these three measures? Why?

You would prefer a higher score. For the all three measures, a higher value indicates a higher fund performance given a measure of risk, which is desirable.

iii. Calculate the Sharpe Ratio, Treynor measure, and Jensen's alpha. Based on your analysis, which fund performs best?

	Jensen's Alpha	SR	Treynor
A	2.8	0.78	10.00
B	2.2	0.63	7.69
C	1.2	0.60	6.67

Based on your analysis, Fund A performs the best as it has the highest score on all three measures.

Part c. (4 points)

You have a time series of realized returns of a mutual fund, a stock and a bond index, and treasuries. You want to perform style analysis of the mutual fund. Give an equation that would allow you to do that and explain its components. Explain how you would estimate its coefficients.

$R_t = alpha + beta_1 Stocks_t + beta_2 Bonds_t + beta_3 Treasuries_t + e_t$

Where R_t is the excess return of the fund, alpha indicates the abnormal return of the fund over the period, and the style funds are the excess returns of the stock and the bond index, as well as the treasuries. The beta coefficients should sum up to one in order to represent the weights in the style funds. The factor exposures give the return due to a style. The factor loadings can be estimated by minimizing the sum of squared residuals (an OLS regression).

Question 3: Fixed Income (15 points)

Part a. (6 points)

Consider the data on the following three coupon bonds:

	Maturity	Coupon	Yield	Face Value
Bond A	2	0.06	0.02	100
Bond B	3	0.05	0.03	100
Bond C	4	0.03	0.04	100

i. Compute the prices, duration and the modified duration of the three bonds.

For bond A:

$P_A = 6/1.02 + 106/1.02^2 = 107.77$
$D_A = 1*6/1.02/107.77 + 2*106/1.02^2/107.77 = 1.95$
$D_A^* = 1.95/(1+0.02) = 1.91$

You can easily follow the steps for the other two bonds

	Price	Duration	Modified Duration
Bond A	107.77	1.95	1.91
Bond B	105.66	2.86	2.78
Bond C	96.37	3.82	3.68

ii. You have a portfolio consisting of a long position in 5 bonds of type B, 4 bonds of type C, and a short position in 2 bonds of type A. Calculate the duration and the modified duration of the portfolio.

Value of the portfolio: -2*Pa+5*Pb+4*Pc = 698.23
Weights of the portfolio: w_A = -2*Pa/698.23 = -0.31. Same principle for the other 2.

	Position	weights
Bond A	-2	-0.31
Bond B	5	0.76
Bond C	4	0.55

Portfolio duration=-wa*Da + wb*Db + wc*Dc=3.68
Portfolio modified duration: D* = wa*D*a + wb*D*b + wc*D*c=3.55

iii. Using duration approximation, what is the change in the value of the portfolio if the yield curve shifts downwards by 100 basis points? And if it shifts downwards by 10 basis points? In which of the two cases the approximation will be more exact and why?

100bp: deltaP/P = -3.55*-0.01 = 0.355
10bp: deltaP/P = 0.035 (more exact as smaller yield change).

Part b. (2 points)

If you are buying a coupon bond between interest paying dates, is the amount you would pay to your broker for the bond more or less than the amount quoted in the financial quotation pages? Discuss the differences and how these differences arise.

If you are buying a bond between interest paying dates, you will pay more than the amount quoted in the financial pages. You will pay that price plus the interest that has accrued since the last interest paying date. That interest belongs to the seller of the bond and will be remitted to the seller by the broker. When the next interest paying date arrives, you will receive the entire coupon payment.

Part c. (4 points)

The zero rates for maturities of 1, 2, and 3 years are given by z1, z2, and z3. Obtain (analytically) the corresponding one-year forward rates for t=1, 2, 3 (f1, f2, f3) in terms of the zero rates. Express the price of a 3-year coupon bond with annual coupon payments of C and face value of 100 using the forward rates.

The forward rates from the zero curve:

$$f_1 = z_1$$

$$f_2 = \frac{(1+z_2)^2}{1+z_1} - 1$$

$$f_3 = \frac{(1+z_3)^3}{(1+z_2)^2} - 1$$

The price of a coupon bond and 3 years maturity using the forward rates:

$$P = \frac{C}{1+f_1} + \frac{C}{(1+f_1)*(1+f_2)} + \frac{C+100}{(1+f_1)*(1+f_2)*(1+f_3)}$$

Part d. (3 points)

Discuss the theories of the term structure of interest rates. Include in your discussion the differences in the theories, and the advantages/disadvantages of each.

The expectations hypothesis is the most commonly accepted theory of term structure. The theory states that the forward rate equals the market consensus expectation of future short-term rates. Thus, yield to maturity is determined solely by current and expected future one-period interest rates. An upward sloping, or normal, yield curve would indicate that investors anticipate an increase in interest rates. An inverted, or downward sloping, yield curve would indicate an expectation of decreased interest rates. A horizontal yield curve would indicate an expectation of no interest rate changes.

The liquidity preference theory of term structure maintains that investors prefer to be liquid to illiquid, all else equal, and will demand a liquidity premium in order to go long term. Thus, liquidity preference readily explains the upward sloping, or normal, yield curve. However, liquidity preference does not readily explain other yield curve shapes.

Question 4: Option Pricing (15 points)

Part a. (2 points)

What is an option hedge ratio? How does the hedge ratio for a call differ from that of a put (or are the two equivalent)? Explain.

An option's hedge ratio is the change in the price of an option for a 1 euro increase in the stock price. A call option has a positive hedge ratio; a put option has a negative hedge ratio. The hedge ratio is the slope of the value function of the option evaluated at the current stock price.

Part b. (7 points)

You are evaluating a stock that is currently selling for $30 per share. Over the investment period of 1 year you think that the stock price might get as low as $25 or as high as $40. There is a call option

available on the stock with an exercise price of $35. The annual interest rate is 6%. Answer the following questions about hedging your position in the stock. Assume that you will buy one share.

i. What is the hedge ratio?
   (5 - 0)/(40 - 25) = 1/3. [If the stock price ends at $40 the call is worth $5; if it ends at $25 the call is worth $0.]
ii. How much would you borrow to purchase the stock? (Hint: make sure the value of your stock portfolio at the end of the holding period is non-negative)
   Borrow the present value of the anticipated minimum stock price = $25/1.06 = $23.58
iii. What is the amount of your net investment in the stock?
   $30 - 23.58 = $6.42
iv. What is the value of the stock at year end, the repayment of the loan, and the total value of your portfolio (all calculated as of the end of the holdings period) under the two scenarios (low and high stock price)?

Scenario	Low Stock Price	High Stock Price
Value of Stock at Year End	$25	$40
Repayment of Loan	-$25	-$25
Total	$0	$15

v. How many call options will you combine with the stock to construct the perfect hedge? Will you buy the calls or sell the calls?

Since the hedge ratio is 1/3 buy one stock and sell three call options.

vi. Show the option values (i.e. the value of your call position) under the two scenarios (low and high stock price).

Scenario	Low Stock Price	High Stock Price
Value of Call Position	$0	$15 {= 3 * $5)

vii. Show the net payoff to your stock, call, and total portfolio under the two scenarios (low and high stock price).

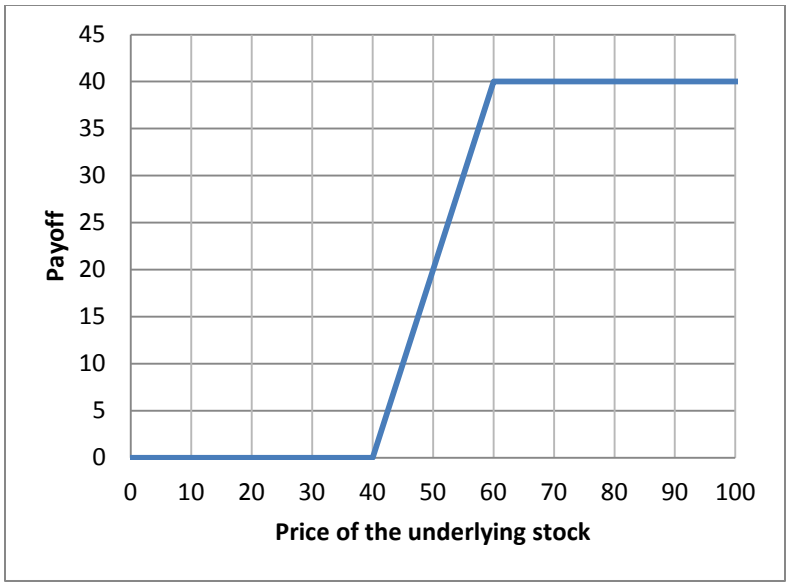
Scenario	Low Stock Price	High Stock Price
Value of Stock at Year End	$25	$40
Value of Call Position	-$0	-$15
Total	$25	$25

viii. What must the price of one call option be?

The value of the stock portfolio equals the value of three calls. The net investment in the stock portfolio is $6.42 so this must equal the value of the three calls. $6.42 = 3C, and C = $2.14.

Part c. (6 points)

The payoff of a collar strategy is given below.



Replicate the payoff using:

i. A combination of call options
Short 2 calls at 60, long 2 calls at 40

ii. A combination of a cash account and put options
Lend PV(40), short 2 puts at 60, long 2 puts at 40