

Faculty of Economics and Business Administration

Exam: Investments 3.4

Code: E_BE3_INV

Coordinator: Dr. Teodor Dyakov

Date: May 21, 2014

Time: 8.45

Duration: 2 hours and 45 minutes

Calculator allowed: Yes

Graphical calculator
allowed: Yes

Number of questions: 20 multiple choice questions and 4 open-ended questions

Type of questions: Open / multiple choice

Answer in: English

Remarks: Be concise and complete in your answers (including calculations). Always explain your answers, even if not explicitly called for. Use your time efficiently, using the maximum number of points per question as a guideline.

Credit score: The maximum possible scores for each part and question are indicated. In total, you can earn 100 points. Your final exam grade is determined by dividing the number of points by 10.

Grades: The grades will be made public on: June 4 2014

Inspection: Tuesday, June 10 2014 at 13.00 in room 3A-31.

Number of pages: (11 (including front page))

Good luck!

PART 1 (MULTIPLE CHOICE; 40 points at maximum)

Read the questions and answers carefully and write down your answer on your answer sheet. Your final score is determined as (# correct answers - 2) * 40/18. Negative scores for this part of the exam are set to zero.

You have been given this probability distribution for the holding-period return for KMP stock:

<u>State of the Economy</u>	<u>Probability</u>	<u>HPR</u>
Boom	.30	18%
Normal growth	.50	12%
Recession	.20	- 5%

1. What is the expected holding-period return for KMP stock?

- A. 10.40%**
- B. 9.32%
- C. 11.63%
- D. 11.54%
- E. 10.88%

$$\text{HPR} = .30 (18\%) + .50 (12\%) + .20 (-5\%) = 10.4\%$$

2. What is the expected standard deviation for KMP stock?

- A. 6.91%
- B. 8.13%**
- C. 7.79%
- D. 7.25%
- E. 8.85%

$$s = [.30 (18 - 10.4)^2 + .50 (12 - 10.4)^2 + .20 (-5 - 10.4)^2]^{1/2} = 8.13\%$$

3. The change from a straight to a kinked capital allocation line is a result of:

- A. reward-to-volatility ratio increasing.
- B. borrowing rate exceeding lending rate.**
- C. an investor's risk tolerance decreasing.
- D. increase in the portfolio proportion of the risk-free asset.
- E. a flawed theory.

The linear capital allocation line assumes that the investor may borrow and lend at the same rate (the risk-free rate), which obviously is not true. Relaxing this assumption and incorporating the higher borrowing rates into the model results in the kinked capital allocation line.

4. Assume that stock market returns do follow a single-index structure. An investment fund analyzes 500 stocks in order to construct a mean-variance efficient portfolio constrained by 500 investments. They will need to calculate _____ estimates of firm-specific variances and _____ estimate/estimates for the variance of the macroeconomic factor.

- A.** 500; 1
- B. 500; 500
- C. 124,750; 1
- D. 124,750; 500
- E. 250,000; 500

For the single-index model, $n(500)$ estimates of firm-specific variances must be calculated and 1 estimate for the variance of the common macroeconomic factor.

5. According to the Capital Asset Pricing Model (CAPM), underpriced securities

- A. have positive betas.
- B. have zero alphas.
- C. have negative betas.
- D.** have positive alphas.
- E. have negative alphas.

According to the Capital Asset Pricing Model (CAPM), underpriced securities have positive alphas.

6. Consider the multifactor APT with two factors. Stock A has an expected return of 17.6%, a beta of 1.45 on factor 1 and a beta of .86 on factor 2. The risk premium on the factor 1 portfolio is 3.2%. The risk-free rate of return is 5%. What is the risk-premium on factor 2 if no arbitrage opportunities exist?

- A.** 9.26%
- B. 3%
- C. 4%
- D. 7.75%
- E. 9.75%

$17.6\% = 1.45(3.2\%) + .86x + 5\%$; $x = 9.26$.

7. Nicholas Manufacturing just announced yesterday that its fourth quarter earnings will be 10% higher than last year's fourth quarter. You observe that Nicholas had an abnormal return of -1.2% yesterday. This suggests that

- A. the market is not efficient.
- B. Nicholas' stock will probably rise in value tomorrow.
- C.** investors expected the earnings increase to be larger than what was actually announced.
- D. investors expected the earnings increase to be smaller than what was actually announced.
- E. earnings are expected to decrease next quarter.

Anticipated earnings changes are impounded into a security's price as soon as expectations are formed. Therefore a negative market response indicates that the earnings surprise was negative, that is, the increase was less than anticipated.

Torque Corporation is expected to pay a dividend of \$1.00 in the upcoming year. Dividends are expected to grow at the rate of 6% per year. The risk-free rate of return is 5% and the expected return on the market portfolio is 13%. The stock of Torque Corporation has a beta of 1.2.

8. What is the intrinsic value of Torque's stock?

- A. \$14.29
- B. \$14.60
- C. \$12.33
- D.** \$11.62
- E. None of these is correct

$$k = 5\% + 1.2(13\% - 5\%) = 14.6\%; P = 1/(\.146 - .06) = \$11.62.$$

9. Suppose two portfolios have the same average return, the same standard deviation of returns, but portfolio A has a lower beta than portfolio B. According to the Treynor measure, the performance of portfolio A _____.

- A.** is better than the performance of portfolio B
- B. is the same as the performance of portfolio B
- C. is poorer than the performance of portfolio B
- D. cannot be measured as there is no data on the alpha of the portfolio
- E. None of these is correct.

The Treynor index is a measure of average portfolio returns (in excess of the risk free return) per unit of systematic risk (as measured by beta).

10. Suppose the risk-free return is 3%. The beta of a managed portfolio is 1.75, the alpha is 0%, and the average return is 16%. Based on Jensen's measure of portfolio performance, you would calculate the return on the market portfolio as

- A. 12.3%
- B. 10.4%**
- C. 15.1%
- D. 16.7%
- E. None of these is correct

$0\% = 16\% - [3\% + 1.75(x - 3\%)]$; $x = 10.4\%$.

11. Bond analysts might be more interested in a bond's yield to call if

- A. the bond's yield to maturity is insufficient.
- B. the firm has called some of its bonds in the past.
- C. the investor only plans to hold the bond until its first call date.
- D. interest rates are expected to rise.
- E. interest rates are expected to fall.**

If interest rates fall the firm is more likely to call the issue and refinance at lower rates.

12. A zero-coupon bond is one that

- A. effectively has a zero percent coupon rate.**
- B. pays interest to the investor based on the general level of interest rates, rather than at a specified coupon rate.
- C. pays interest to the investor without requiring the actual coupon to be mailed to the corporation.
- D. is issued by state governments because they don't have to pay interest.
- E. is analyzed primarily by focusing ("zeroing in") on the coupon rate.

Zero-coupon bonds pay no interest. Investors receive the face value at maturity.

13. What is the price of a 4-year maturity bond with a 12% coupon rate paid annually? (Par value = \$1,000)

- A. \$742.09
- B. \$1,222.09
- C. \$1,000.00
- D. \$1,141.92
- E. None of these is correct.

It looks like some data is missing from this question. Thus, I accept any answer given (or not given). You will further get 5 points for the time lost on this question. Thus, by accident, this exam has a very similar mistake to the one in March and you get the exact same compensation for the mistake.

14. Which of the following combinations will result in a sharply increasing yield curve?

- A. Increasing future expected short rates and increasing liquidity premiums
- B. Decreasing future expected short rates and increasing liquidity premiums
- C. Increasing future expected short rates and decreasing liquidity premiums
- D. Increasing future expected short rates and constant liquidity premiums
- E. Constant future expected short rates and increasing liquidity premiums

Both of the forces will act to increase the slope of the yield curve.

15. Which of the following is **not** true?

- A. Holding other things constant, the duration of a bond increases with time to maturity.
- B. Given time to maturity, the duration of a zero-coupon decreases with yield to maturity.
- C. Given time to maturity and yield to maturity, the duration of a bond is higher when the coupon rate is lower.
- D. Duration is a better measure of price sensitivity to interest rate changes than is time to maturity.
- E. All of these are correct.

The duration of a zero-coupon bond is equal to time to maturity, and is independent of yield to maturity.

16. According to the put-call parity theorem, the value of a European put option on a non-dividend paying stock is equal to:

- A. the call value plus the present value of the exercise price plus the stock price.
- B. the call value plus the present value of the exercise price minus the stock price.
- C. the present value of the stock price minus the exercise price minus the call price.
- D. the present value of the stock price plus the exercise price minus the call price.
- E. None of these is correct.

$P = C - SO + PV(X) + PV(\text{dividends})$, where SO = the market price of the stock, and X = the exercise price.

17. All the inputs in the Black-Scholes Option Pricing Model are directly observable **except**

- A. the price of the underlying security.
- B. the risk free rate of interest.
- C. the time to expiration.
- D. the variance of returns of the underlying asset return.
- E. None of these is correct.

The variance of the returns of the underlying asset is not directly observable, but must be estimated from historical data, from scenario analysis, or from the prices of other options.

18. Portfolio A consists of 500 shares of stock and 500 calls on that stock. Portfolio B consists of 800 shares of stock. The call delta is 0.6. Which portfolio has a higher dollar exposure to a change in stock price?

- A. Portfolio B.
- B. Portfolio A.
- C. The two portfolios have the same exposure.
- D. A if the stock price increases and B if it decreases.
- E. B if the stock price decreases and A if it increases.

500 calls $(0.6) = 300$ shares + 500 shares = 800 shares; 800 shares = 800 shares.

19. On January 1, the listed spot and futures prices of a Treasury bond were 93.8 and 93.13. You purchased \$100,000 par value Treasury bonds and sold one Treasury bond futures contract. One month later, the listed spot price and futures prices were 94 and 94.09, respectively. If you were to liquidate your position, your profits would be

- A. \$125 loss.
- B. \$125 profit.
- C. \$12.50 loss.
- D. \$1,250 loss.
- E. None of these is correct.

On bonds: $\$94,000 - \$93,250 = \$750$; On futures: $\$93,406.25 - \$94,281.25 = -\$875$; Net profits: $\$750 - \$875 = -\$125$.

20. Suppose that the risk-free rates in the United States and in the United Kingdom are 4% and 6%, respectively. The spot exchange rate between the dollar and the pound is \$1.60/BP. What should the futures price of the pound for a one-year contract be to prevent arbitrage opportunities, ignoring transactions costs?

- A. \$1.60/BP
- B. \$1.70/BP
- C. \$1.66/BP
- D. \$1.63/BP
- E. \$1.57/BP

$\$1.60(1.04/1.06) = \$1.57/\text{BP}$.

PART 2 (OPEN QUESTIONS; 60 points at maximum)

Question 1: Equilibrium Pricing Models (15 points)

Part a: 4 points

Give the formula of the CAPM and explain its notation. What are the assumptions underlying the CAPM? How do they relate to empirical evidence?

The assumptions are:

- (a) The market is composed of many small investors, who are price-takers; i. e., perfect competition. In reality this assumption was fairly realistic until recent years when institutional investors increasingly began to influence the market with their large transactions.
- (b) All investors have the same holding period. Obviously, different investors have different goals, and thus have different holding periods.
- (c) Investments are limited to those that are publicly traded. In addition, it is assumed that investors may borrow or lend any amount at a fixed, risk-free rate. Obviously, investors may purchase assets that are not publicly traded; however, the dollar volume of publicly traded assets is considerable. The assumption that investors can borrow or lend any amount at a fixed, risk-free rate obviously is false. However, the model can be modified to incorporate different borrowing and lending rates.
- (d) Investors pay no taxes on returns and incur no transaction costs. Obviously, investors do pay taxes and do incur transaction costs.
- (e) All investors are mean-variance efficient. This assumption implies that all investors make decisions based on maximizing returns available at an acceptable risk level; most investors probably make decisions in this manner. However, some investors are pure wealth maximizers (regardless of the risk level); and other investors are so risk averse that avoiding risk is their only goal.
- (f) All investors have homogeneous expectations, meaning that given the same data all investors would process the data in the same manner, resulting in the same risk/return assessments for all investment alternatives.

Part b: 2 points

Security A has a beta of 1.0 and an expected return of 12%. Security B has a beta of 0.75 and an expected return of 11%. The risk-free rate is 6%. Explain the arbitrage opportunity that exists; explain how an investor can take advantage of it. Give specific details about how to form the portfolio, what to buy and what to sell.

An arbitrage opportunity exists because it is possible to form a portfolio of security A and the risk-free asset that has a beta of 0.75 and a different expected return than security B. The investor can accomplish this by choosing .75 as the weight in A and .25 in the risk-free asset. This portfolio would have $E(r_p) = 0.75(12\%) + 0.25(6\%) = 10.5\%$, which is less than B's 11% expected return. The investor should buy B and finance the purchase by short selling A and borrowing at the risk-free asset.

Part c: 4 points

Assume that the CAPM holds. The risk free rate r_f is 5% and the market return r_M is 8%. Consider the following two stocks:

Stocks	β	P/E
A	1.5	14.29
B	0.8	13.51

β - market beta of each stock

P/E – price-earnings ratio

The retention ratio (**b**) for the two stocks is 0.5.

i. Compute the return on equity (ROE) of the two stocks, assuming a constant growth model with endogenous earnings growth (round at the third decimal).

$$k_A = 0.05 + 1.5 \cdot (0.08 - 0.05) = 0.095, \quad k_B = 0.05 + 0.8 \cdot (0.08 - 0.05) = 0.074$$

$$ROE_A = (14.29 \cdot 0.095 - 0.5) / (14.29 \cdot 0.5) = 0.12$$

$$ROE_B = (13.51 \cdot 0.074 - 0.5) / (13.51 \cdot 0.5) = 0.074$$

ii. Estimate the growth rate for both firms.

$$g_A = 0.12 \cdot 0.5 = 0.06, \quad g_B = 0.074 \cdot 0.5 = 0.037$$

iii. What would the effect on the P/E ratio be if you change the retention ratio of stock B, keeping ROE constant? Why?

It will not change since $ROE_B = k_B$.

Part d: 5 points

Consider the multifactor APT. There are three independent economic factors, F_1 , F_2 , and F_3 . The risk-free rate of return is 5%. The following information is available about three well-diversified portfolios A, B and C:

Portfolio	beta on F_1	beta on F_2	beta on F_3	expected return	variance
A	0.5	1	1	30%	40%
B	0	1	0.5	15%	20%
C	0	0	1	10%	15%

Assuming no arbitrage opportunities exist, calculate the risk premia on the three factor Portfolios

The risk premia of the three factors should solve the following system of equations

$$30\% = 5\% + 0.5 \cdot RP_1 + 1 \cdot RP_2 + 1 \cdot RP_3$$

$$15\% = 5\% + 0 \cdot RP_1 + 1 \cdot RP_2 + 0.5 \cdot RP_3$$

$$10\% = 5\% + 0 \cdot RP_1 + 0 \cdot RP_2 + 1 \cdot RP_3$$

It follows that $RP_1=25\%$, $RP_2=7.5\%$, $RP_3=5\%$

Give an expression for the variance of a well-diversified portfolio along the lines of the multifactor APT. Calculate the variance of the two factors, given the data above.

Because A, B, and C are well-diversified:

$$\begin{aligned}\beta_{A1}^2 \sigma_{F1}^2 + \beta_{A2}^2 \sigma_{F2}^2 + \beta_{A3}^2 \sigma_{F3}^2 &= \sigma_A^2 \\ \beta_{B1}^2 \sigma_{F1}^2 + \beta_{B2}^2 \sigma_{F2}^2 + \beta_{B3}^2 \sigma_{F3}^2 &= \sigma_B^2 \\ \beta_{C1}^2 \sigma_{F1}^2 + \beta_{C2}^2 \sigma_{F2}^2 + \beta_{C3}^2 \sigma_{F3}^2 &= \sigma_C^2\end{aligned}$$

Thus,

$$\begin{aligned}0.25\sigma_{F1}^2 + 1\sigma_{F2}^2 + 1\sigma_{F3}^2 &= 40\% \\ 1\sigma_{F2}^2 + 0.25\sigma_{F3}^2 &= 20\% \\ 1\sigma_{F3}^2 &= 15\%\end{aligned}$$

It follows that $\sigma_{F1}^2 = 1.65$, $\sigma_{F2}^2 = 0.1625$, and $\sigma_{F3}^2 = 0.15$

Construct a portfolio of A, B, and C that has exposure of 1 to F1 and 1.5 to F2. What are the weights of the three portfolios A, B, and C? What is its exposure to F3?

The weights of the A, B, and C should satisfy the following system of equations:

$$\begin{aligned}\omega_1 + \omega_2 + \omega_3 &= 1 \\ \beta_1 \omega_1 + \beta_2 \omega_2 + \beta_3 \omega_3 &= 1 \\ \beta_2 \omega_1 + \beta_2 \omega_2 + \beta_2 \omega_3 &= 1.5\end{aligned}$$

Thus,

$$\begin{aligned}\omega_1 + \omega_2 + \omega_3 &= 1 \\ 0.5\omega_1 &= 1 \\ 1\omega_1 + 1\omega_2 &= 1.5\end{aligned}$$

It follows that $\omega_1 = 2$, $\omega_2 = -0.5$, and $\omega_3 = -0.5$

It is further easy to see that the exposure of the combined portfolio to F3 is 1.25

Question 2: Portfolio Construction and Performance Measurement (15 points)

Part a: 2 points

Describe how an investor may combine a risk-free asset and one risky asset in order to obtain the optimal portfolio for that investor.

The investor may combine a risk-free asset (U.S. T-bills or a money market mutual fund) and a risky asset, such as an indexed mutual fund in the proper portions to obtain the desired risk-return relationship for that investor. The investor must realize that the risk-return relationship is a linear one, and that in order to earn a higher return, the investor must be willing to assume more risk. The investor must first determine the amount of risk that he or she can tolerate (in terms of the standard deviation of the total portfolio, which is the product of the proportion of total assets invested in the risky asset and the standard deviation of the risky asset). One minus this weight is the proportion of total assets to be invested in the risk-free asset. The portfolio return is the weighted averages of the returns on the two respective assets. Such an asset allocation plan is probably the easiest, most efficient, and least expensive for the individual investor to build an optimal portfolio.

Part b: 5 points

You are evaluating two investment alternatives. One is a passive market portfolio with an expected return of 10% and a standard deviation of 16%. The other is a fund that is actively managed by your broker. This fund has an expected return of 15% and a standard deviation of 20%. The risk-free rate is currently 7%. Answer the questions below based on this information.

- What is the slope of the Capital Market Line?
- What is the slope of the Capital Allocation Line offered by your broker's fund?
- Draw the CML and the CAL on one graph.
- What is the maximum fee your broker could charge and still leave you as well off as if you had invested in the passive market fund? (Assume that the fee would be a percentage of the investment in the broker's fund, and would be deducted at the end of the year.)
- How would it affect the graph if the broker were to charge the full amount of the fee?

The slope of the CML is $(10 - 7)/16 = 0.1875$.

The slope of the CAL is $(15 - 7)/20 = 0.40$.

On the graph, both the CML and the CAL have an intercept equal to the risk-free rate (7%). The CAL, with a slope of 0.40, is steeper than the CML, with a slope of 0.1875.

To find the maximum fee the broker can charge, the equation $(15 - 7 - \text{fee})/20 = 0.1875$ is solved for "fee".

The resulting fee is 4.25%.

If the broker charges the full amount of the fee, the CAL's slope would also be 0.1875, so it would rotate down and be identical to the CML.

Part c. (5 points)

You want to evaluate three mutual funds, based on the information below. The market return is 16% and the risk free rate is 6%.

	Average Return	Standard Deviation	Beta
Fund A	0.24	0.3	1.5
Fund B	0.14	0.1	0.5
Fund C	0.22	0.2	1

Provide the formulas for the Sharpe Ratio, Treynor measure, and Jensen's alpha.

Sharpe ratio = (return of the portfolio – risk free rate) / std. deviation

Treynor's measure = (return of the portfolio – risk free rate) / beta

Jensen's alpha = return of the portfolio – (risk free rate + beta * (return of the market – risk free rate))

How are the Sharpe Ratio and Treynor's measure related to Jensen's alpha? Comment briefly.

Both the Sharpe Ratio and Treynor's measure are increasing in alpha.

Calculate the Sharpe Ratio, Treynor measure, and Jensen's alpha. Based on your analysis, which fund performs best?

	Sharpe Ratio	Treynor	Jensen
Fund A	0.6	0.12	0.03
Fund B	0.8	0.16	0.03
Fund C	0.8	0.16	0.06

Based on your analysis, Fund C performs best as it has the highest score on all three measures.

Part d. (3 points)

Draw a graph of a typical efficient frontier. Explain why the efficient frontier is shaped the way it is.

The efficient frontier has a curved appearance, as shown throughout the course. The typical shape results from the fact that assets' returns are not perfectly (positively or negatively) correlated.

Question 3: Fixed Income (15 points)

Part a. (3 points)

You hold a \$50 million portfolio of par value bonds with a coupon rate of 10 percent paid annually and 15 years to maturity. How many T-bond futures contracts do you need to hedge the portfolio against an unanticipated change in the interest rate of 0.18%? Assume the market interest rate is 10 percent and that T-bond futures contracts call for delivery of an 8 percent coupon (paid annually), 20-year maturity T-bond.

398 contracts short

Here are the steps to obtain the answer:

$0.9864485 \times \$50 \text{ M} = \$49,322,425$; $\$50,000,000 - \$49,322,425 = \$677,575$ loss on bonds;

$\$100.00 - \$82.97 = \$17.03 \times 100 = \1703 gain on futures;

The loss on bonds needs to equal the gain on the future, thus $\$677,575 / \$1,703 = 398$ contracts **short**.

Part b. (3 points)

Why are many bonds callable? What is the disadvantage to the investor of a callable bond? What does the investor receive in exchange for a bond being callable? How are bond valuation calculations affected if bonds are callable?

Many bonds are callable to give the issuer the option of calling the bond in and refunding (reissuing) the bond if interest rates decline. Bonds issued in a high interest rate environment will have the call feature. Interest rates must decline enough to offset the cost of floating a new issue. The disadvantage to the investor is that the investor will not receive that long stream of constant income that the bondholder would have received with a noncallable bond. In return, the yields on callable bonds are usually slightly higher than the yields on noncallable bonds of equivalent risk. When the bond is called, the investor receives the call price (an amount greater than par value). The bond valuation calculation should include the call price rather than the par value as the final amount received; also, only the cash flows until the first call should be discounted. The result is that the investor should be looking at yield to first call, not yield to maturity, for callable bonds.

Part c. (4 points)

Consider the data on the following three coupon bonds:

	Maturity	Coupon	Yield	Face Value
Bond A	2	0.07	0.02	100
Bond B	3	0.05	0.04	100
Bond C	4	0.04	0.03	100

Compute the prices, duration and the modified duration of the three bonds.

For bond A:

$$P_A = 7/1.02 + 107/1.02^2 = 109.71$$

$$D_A = 1 \cdot 7/1.02/109.71 + 2 \cdot 107/1.02^2/109.71 = 1.94$$

$$D_A^* = 1.94/(1+0.02) = 1.90$$

You can easily follow the steps for the other two bonds

	Price	Duration	Modified Duration
Bond A	109.71	1.94	1.90
Bond B	102.78	2.86	2.75
Bond C	103.72	3.78	3.67

You have a portfolio consisting of a long position in 4 bonds of type B, 5 bonds of type C, and a short position in 1 bonds of type A. Calculate the duration and the modified duration of the portfolio.

$$\text{Value of the portfolio: } -1 \cdot P_A + 4 \cdot P_B + 5 \cdot P_C = 819.98$$

$$\text{Weights of the portfolio: } w_A = -1 \cdot P_A / 819.98 = -0.13. \text{ Same principle for the other 2.}$$

	Position	weights
Bond A	-1	-0.13
Bond B	4	0.50
Bond C	5	0.63

$$\text{Portfolio duration} = -w_A \cdot D_A + w_B \cdot D_B + w_C \cdot D_C = 3.57$$

$$\text{Portfolio modified duration: } D^* = w_A \cdot D_A^* + w_B \cdot D_B^* + w_C \cdot D_C^* = 3.45$$

Using duration approximation, what is the change in the value of the portfolio if the yield curve shifts upwards by 100 basis points? And if it shifts upwards by 10 basis points? In which of the two cases the approximation will be more exact and why?

$$100\text{bp: } \Delta P/P = -3.45 \cdot 0.01 = -0.345$$

$$10\text{bp: } \Delta P/P = -0.035 \text{ (more exact as smaller yield change).}$$

Part d. (3 points)

You hold a bond portfolio with a duration D and convexity C . The current zero curve is flat at 5%. Construct a portfolio of zero coupon bonds with maturities of 1, 2 and 3 years that match the duration and the convexity of your portfolio. Solve analytically for the weights of the new zero coupon bond portfolio.

Denote by a , b , and c the weights of the 3 zero coupon bonds in the portfolio. They solve the following system:

$$a+b+c = 1$$

$$a+2b+3c = D$$

$$2a/(1+0.05)^2 + 6b/(1+0.05)^2 + 12c/(1+0.05)^2 = C$$

Solving it yields:

$$a = 3-3D+1/2C(1+0.05)^2$$

$$b = 5D - 3 - C(1+0.05)^2$$

$$c = 1/2C(1+0.05)^2 - 2D + 1.$$

Part e. (2 points)

Although the expectations of increases in future interest rates can result in an upward sloping yield curve; an upward sloping yield curve does not in and of itself imply the expectations of higher future interest rates. Explain.

The effects of possible liquidity premiums confound any simple attempt to extract expectation from the term structure. That is, the upward sloping yield curve may be due to expectations of interest rate increases, or due to the requirement of a liquidity premium, or both. The liquidity premium could more than offset expectations of decreased interest rates, and an upward sloping yield would result.

Question 4: Option Pricing (15 points)

Part a. (7 points)

You are evaluating a stock that is currently selling for 60 per share. Over the investment period of 1 year you think that the stock price might get as low as 50 or as high as 80. There is a call option available on the stock with an exercise price of 70. The annual interest rate is 8%. Answer the following questions about hedging your position in the stock. Assume that you will buy one share.

- i. What is the hedge ratio (Hint: use the option pay-offs and the stock prices for the high and the low scenarios)?

$$(10 - 0)/(80 - 50) = 1/3. \text{ [If the stock price ends at 80 the call is worth 10; if it ends at 50 the call is worth 0.]}$$

- ii. How much would you borrow to purchase the stock? (Hint: make sure the value of your stock portfolio at the end of the holding period is non-negative)

Borrow the present value of the anticipated minimum stock price = $\$50/1.08 = 46.30$

- iii. What is the amount of your net investment in the stock?

$60 - 46.30 = 13.70$

- iv. Complete the table below to show the value of your stock portfolio at the end of the holding period.

	Low Stock Price	High Stock Price
Value of Stock at Year End	50	80
Repayment of Loan	-50	-50
Total	0	30

- v. How many call options will you combine with the stock to construct the perfect hedge? Will you buy the calls or sell the calls?

Since the hedge ratio is $1/3$ buy one stock and sell three call options.

- vi. Show the option values in the table below.

	Low Stock Price	High Stock Price
Value of Call Position	0	30

- vii. Show the net payoff to your portfolio in the table below.

	Low Stock Price	High Stock Price
Value of Stock at Year End	50	80
Value of Call Position	0	-30
Total	50	50

viii. What must the price of one call option be?

The value of the stock portfolio equals the value of three calls. The net investment in the stock portfolio is 13.70 so this must equal the value of the three calls. $13.70 = 3C$, and $C = 4.57$.

Part b. (3 points)

Discuss the relationship between option prices and time to expiration, volatility of the underlying stocks, and the exercise price.

The longer the time to expiration, the higher the premium because it is more likely that an option will become more valuable (more time for the stock price to change). The greater the volatility of the underlying stock, the greater the option premium; the more volatile the stock, the more likely it is that the option will become more valuable (e. g., move from an out of the money to an in the money option, or become more in the money). For call options, the lower the exercise price, the more valuable the option, as the option owner can buy the stock at a lower price. For a put option, the lower the exercise price, the less valuable the option, as the owner of the option may be required to sell the stock at a lower than market price.

Part c. (5 points)

Construct an options strategy that allows an investor to benefit from large upward or downward price moves of the underlying asset, while limiting his losses otherwise. Assume that the current price is 40 and that the investor aims at limiting his loss to 2 when the price of the underlying moves by 5% in either direction. For higher price volatility the investor wants to assure an upward potential. The premium for both a put and a call is 1.

What options will you use to construct such a strategy? Specify type and strike price.

Draw a profit diagram of the option strategy, considering the premium paid or received for the options.

Long one put at 38, long one call at 42

