

Faculty of Economics and Business Administration

Exam: Investments 3.4

Code: E_BE3_INV

Coordinator: Dr. Teodor Dyakov

Date: March 24, 2014

Time: 8.30

Duration: 2 hours and 45 minutes

Calculator allowed: Yes

Graphical calculator
allowed: Yes

Number of questions: 20 multiple choice questions and 4 open-ended questions

Type of questions: Open/ multiple choice

Answer in: English

Remarks: Be concise and complete in your answers (including calculations). Always explain your answers, even if not explicitly called for. Use your time efficiently, using the maximum number of points per question as a guideline.

Credit score: The maximum possible scores for each part and question are indicated. In total, you can earn 100 points. Your final exam grade is determined by dividing the number of points by 10.

Grades: The grades will be made public on: April 7 2014

Inspection: Tuesday, April 8 2014 at 13.00 in room 3A-31.

Number of pages: (11 (including front page))

Good luck!

PART 1 (MULTIPLE CHOICE; 40 points at maximum)

Read the questions and answers carefully and write down your answer on your answer sheet. Your final score is determined as (# correct answers - 2) * 40/18. Negative scores for this part of the exam are set to zero.

1. You purchase a share of "Company X" stock for €90. One year later, after receiving a dividend of €3, you sell the stock for €92. What is your holding-period return?

- A. 4.44%
- B. 2.22%
- C. 3.33%
- D. 5.56%**
- E. 5.91%

$$\text{HPR} = (92 - 90 + 3)/90 = 5.56\%.$$

You invest \$100 in a risky asset with an expected rate of return of 0.11 and a standard deviation of 0.21 and a T-bill with a rate of return of 0.045.

2. A portfolio that has an expected outcome of \$114 is formed by

- A. Investing \$100 in the risky asset.
- B. Investing \$80 in the risky asset and \$20 in the risk-free asset.
- C. Borrowing \$46 at the risk-free rate and investing the total amount (\$146) in the risky asset.
- D. Investing \$43 in the risky asset and \$57 in the riskless asset.
- E. Such a portfolio cannot be formed.

WRONG: NO DATA WAS GIVEN, YOU GET A POINT FOR IT NO MATTER WHAT YOU INDICATED ON YOUR EXAMINATION SHEETS!

3. Suppose you held a well-diversified portfolio with a very large number of securities, and that the single index model holds. If the σ of your portfolio was 0.18 and σ_M was 0.24, the β of the portfolio would be approximately _____.

- A. 0.75**
- B. 0.56
- C. 0.07
- D. 1.03
- E. 0.86

$$s_p^2/s_m^2 = b^2; (0.18)^2/(0.24)^2 = 0.5625; b = 0.75.$$

4. The market risk, beta, of a security is equal to

A. the covariance between the security's return and the market return divided by the variance of the market's returns.

- B. the covariance between the security and market returns divided by the standard deviation of the market's returns.
- C. the variance of the security's returns divided by the covariance between the security and market returns.
- D. the variance of the security's returns divided by the variance of the market's returns.
- E. the variance of the security's return divided by the standard deviation of the market's returns.

Beta is a measure of how a security's return covaries with the market returns, normalized by the market variance.

5. Imposing the no-arbitrage condition on a single-factor security market implies which of the following statements?

- I) the expected return-beta relationship is maintained for all but a small number of well-diversified portfolios.
 - II) the expected return-beta relationship is maintained for all well-diversified portfolios.
 - III) the expected return-beta relationship is maintained for all but a small number of individual securities.
 - IV) the expected return-beta relationship is maintained for all individual securities.
- A. I and III are correct.
 - B. I and IV are correct.
 - C. II and III are correct.
 - D. II and IV are correct.
 - E. Only I is correct.

The expected return-beta relationship must hold for all well-diversified portfolios and for all but a few individual securities; otherwise arbitrage opportunities will be available.

6. Music Doctors has a beta of 2.25. The annualized market return yesterday was 12%, and the risk-free rate is currently 4%. You observe that Music Doctors had an annualized return yesterday of 15%.

Assuming that markets are efficient, this suggests that

- A. bad news about Music Doctors was announced yesterday.
- B. good news about Music Doctors was announced yesterday.
- C. no news about Music Doctors was announced yesterday.
- D. interest rates rose yesterday.
- E. interest rates fell yesterday.

$AR = 15\% - (4\% + 2.25 (8\%)) = -7.0\%$. A negative abnormal return suggests that there was firm-specific bad news.

7. Old Quartz Gold Mining Company is expected to pay a dividend of \$8 in the coming year. Dividends are expected to decline at the rate of 2% per year. The risk-free rate of return is 6% and the expected return on the market portfolio is 14%. The stock of Old Quartz Gold Mining Company has a beta of -0.25. The intrinsic value of the stock is _____.

- A. \$80.00
- B. \$133.33**
- C. \$200.00
- D. \$400.00
- E. None of these is correct

$$k = 6\% + [-0.25(14\% - 6\%)] = 4\%; P = 8/[.04 - (-.02)] = \$133.33.$$

8. The P/E ratio that is based on a firm's financial statements and reported in the newspaper stock listings is different from the P/E ratio derived from the dividend discount model (DDM) because

- A. the DDM uses a different price in the numerator.
- B. the DDM uses different earnings measures in the denominator.**
- C. the prices reported are not accurate.
- D. the people who construct the ratio from financial statements have inside information.
- E. They are not different - this is a "trick" question.

Both ratios use the same numerator - the market price of the stock. But P/Es from financial statements use the most recent past accounting earnings, while the DDM uses expected future economic earnings.

9. You want to evaluate three mutual funds using the Treynor measure for performance evaluation. The risk-free return during the sample period is 6%. The average returns, standard deviations, and betas for the three funds are given below, in addition to information regarding the S&P 500 index.

	Average Return	Standard. Deviation	Beta
Fund A	13%	10%	0.5
Fund B	19%	20%	1.0
Fund C	25%	30%	1.5
S&P 500	18%	16%	1.0

The fund with the highest Treynor measure is _____.

- A. Fund A**
- B. Fund B
- C. Fund C
- D. Funds A and B are tied for highest
- E. Funds A and C are tied for highest

$$A: (13\% - 6\%)/0.5 = 14; B: (19\% - 6\%)/1.0 = 13; C: (25\% - 6\%)/1.5 = 12.7; S\&P\ 500: (18\% - 6\%)/1.0 = 12.$$

10. You purchased an annual interest coupon bond one year ago that had 6 years remaining to maturity at that time. The coupon interest rate was 10% and the par value was \$1,000. At the time you purchased the bond, the yield to maturity was 8%. If you sold the bond after receiving the first interest payment and the yield to maturity continued to be 8%, your annual total rate of return on holding the bond for that year would have been _____.

- A. 7.00%
- B. 7.82%
- C. 8.00%**
- D. 11.95%
- E. None of these is correct.

$FV = 1000, PMT = 100, n = 6, i = 8, PV = 1092.46; FV = 1000, PMT = 100, n = 5, i = 8, PV = 1079.85; HPR = (1079.85 - 1092.46 + 100)/1092.46 = 8\%$

11. An upward sloping yield curve

- A. may be an indication that interest rates are expected to increase.
- B. may incorporate a liquidity premium.
- C. may reflect the confounding of the liquidity premium with interest rate expectations.
- D. All of these are correct.**
- E. A and B are correct

One of the problems of the most commonly used explanation of term structure, the expectations hypothesis, is that it is difficult to separate out the liquidity premium from interest rate expectations.

Year	1-Year Forward Rate
1	5.8%
2	6.4%
3	7.1%
4	7.3%
5	7.4%

12. What should the purchase price of a 2-year zero coupon bond be if it is purchased at the beginning of year 2 and has face value of \$1,000?

- A. \$877.54**
- B. \$888.33
- C. \$883.32
- D. \$893.36
- E. \$871.80

$$\$1,000/[(1.064)(1.071)] = \$877.54$$

13. The duration of a 5-year zero-coupon bond is

- A. smaller than 5.
- B. larger than 5.
- C. equal to 5.**
- D. equal to that of a 5-year 10% coupon bond.
- E. None of these is correct.

Duration of a zero-coupon bond equals the bond's maturity.

14. When interest rates decline, the duration of a 10-year bond selling at a premium

- A. increases.**
- B. decreases.
- C. remains the same.
- D. increases at first, then declines.
- E. decreases at first, then increases.

The relationship between interest rates and duration is an inverse one.

15. A European put option allows the holder to

- A. buy the underlying asset at the striking price on or before the expiration date.
- B. sell the underlying asset at the striking price on or before the expiration date.
- C. potentially benefit from a stock price increase.
- D. sell the underlying asset at the striking price on the expiration date.**
- E. potentially benefit from a stock price increase and sell the underlying asset at the striking price on the expiration date.

A European put option allows the buyer to sell the underlying asset at the striking price only on the expiration date. The put option also allows the investor to benefit from an expected stock price decrease while risking only the amount invested in the contract.

16. Top Flight Stock currently sells for \$53. A one-year call option with strike price of \$58 sells for \$10, and the risk free interest rate is 5.5%. What is the price of a one-year put with strike price of \$58?

- A. \$10.00
- B. \$12.12
- C. \$16.00
- D. \$11.97**
- E. \$14.13

$$P = 10 - 53 + 58/(1.055); P = 11.97$$

17. Other things equal, the price of a stock call option is positively correlated with the following factors **except**

- A. the stock price.
- B. the time to expiration.
- C. the stock volatility.
- D. the exercise price.**
- E. None of these is correct.

The exercise price is negatively correlated with the call option price.

18. Portfolio A consists of 150 shares of stock and 300 calls on that stock. Portfolio B consists of 575 shares of stock. The call delta is 0.7. Which portfolio has a higher dollar exposure to a change in stock price?

- A. Portfolio B.**
- B. Portfolio A.
- C. The two portfolios have the same exposure.
- D. A if the stock price increases and B if it decreases.
- E. B if the stock price decreases and A if it increases.

$$300 \text{ calls } (0.7) = 210 \text{ shares} + 150 \text{ shares} = 360 \text{ shares}; 575 \text{ shares} = 575 \text{ shares.}$$

19. Which one of the following statements regarding "basis" is **not** true?

- A. The basis is the difference between the futures price and the spot price.
- B. The basis risk is borne by the hedger.
- C. A short hedger suffers losses when the basis decreases.**
- D. The basis increases when the futures price increases by more than the spot price.
- E. None of these is true.

If you think one asset is overpriced relative to another, you sell the overpriced asset and buy the other one.

20. One reason swaps are desirable is that
- A. they are free of credit risk.
 - B. they have no transactions costs.
 - C. they increase interest rate volatility.
 - D. they increase interest rate risk.
 - E. they offer participants easy ways to restructure their balance sheets.

For example, a firm can change a floating-rate obligation into a fixed-rate obligation and vice versa.

PART 2 (OPEN QUESTIONS; 60 points at maximum)

Question 1: Equilibrium Pricing Models (15 points)

Part a. (2 points)

Explain the separation property of a portfolio selection problem.

The portfolio choice problem may be separated into two independent tasks: determination of the optimal portfolio that does not depend on individual investor's preferences and allocation of the portfolio between risk-free and risky assets, which depends on the risk aversion of the investor.

Part b. (2 points)

Give the equation of the single factor model. Explain the decomposition of the risky asset's return and of its variance.

The risky asset's return can be decomposed into an expected and an unexpected part: $r_i = E(r_i) + \beta_i m + e_i$, where m is a common factor and e_i is the unexpected return.

Similarly, the variance of the risky asset has two components – systematic and firm specific: $\sigma_i^2 = \beta_i^2 \sigma_m^2 + \sigma_{e_i}^2$, where σ_m^2 is the variance of the common factor and $\sigma_{e_i}^2$ is the variance of the firm-specific factor.

Part c. (5 points)

Assume that the CAPM holds. The risk free rate r_f is 3% and the market return r_M is 5%. Consider the following two stocks:

Stocks	β	P/E
A	1.3	15.15
B	1.1	19.23

β - market beta of each stock
P/E – price-earnings ratio

The retention ratio (**b**) for the two stocks is 0.5.

- i. Compute the return on equity (ROE) of the two stocks, assuming a constant growth model with endogenous earnings growth (round at the third decimal).

$$k_A = 0.03 + 1.3 \times (0.05 - 0.03) = 0.056, k_B = 0.03 + 1.1 \times (0.05 - 0.03) = 0.052$$

$$ROE_A = (15.15 \times 0.056 - 0.5) / (15.15 \times 0.5) = 0.046, ROE_B = (19.23 \times 0.052 - 0.5) / (19.23 \times 0.5) = 0.052$$

- ii. What value for b will make the P/E ratio the highest possible for stock A, if the ROE is kept at the level calculated in (i)?

b close to 0, as for stock A $ROE < k$

- iii. What would the effect on the P/E ratio be if you change the retention ratio of stock B, keeping ROE constant? Why?

The P/E ratio will remain constant for changing b, as $ROE = k$ for stock B.

Part d. (2 points)

Consider the multifactor APT. There are two independent economic factors, F1 and F2. The risk-free rate of return is 6%. The following information is available about two well-diversified portfolios:

Portfolio	B on F_1	β on F_2	Expected Return
A	1.0	2.0	19%
B	2.0	0.0	12%

Assume that no arbitrage opportunities exist. Compute the risk premium on factor F_1 and the risk premium on factor F_2 .

$$2A: 38\% = 12\% + 2.0(RP1) + 4.0(RP2); B: 12\% = 6\% + 2.0(RP1) + 0.0(RP2); 26\% = 6\% + 4.0(RP2); RP2 = 5\%; A: 19\% = 6\% + RP1 + 2.0(5); RP1 = 3\%.$$

Part e. (4 points)

Consider the Carhart Four-factor Model, which is an extension of the Fama-French 3-factor model:

$$r_i = \alpha_i + \beta_i r_M + \gamma_i SMB + \delta_i HML + \kappa_i MOM + e_i$$

where r_i is the return of a stock i , r_M is the market return, SMB is a factor that proxies for size, HML – for value, and MOM – for momentum. The variances of the four factors are respectively σ_M^2 , σ_{SMB}^2 , σ_{HML}^2 ,

and σ_{MOM}^2 . The variance of the idiosyncratic source of risk is $\sigma^2(e)$ for all stocks i . Assume that the idiosyncratic sources of risk are uncorrelated, and that the factors are uncorrelated as well.

Now consider two stocks ($i=\{1, 2\}$).

- i. Give an expression for the systematic risk of each of the two stocks.

$$\sigma_i^2 = \beta_i^2 \sigma_M^2 + \gamma_i^2 \sigma_{SMB}^2 + \delta_i^2 \sigma_{HML}^2 + \kappa_i^2 \sigma_{MOM}^2$$

- ii. Construct an equally weighted portfolio of the two stocks. What is its non-systematic risk component? Compare it to the non-systematic risk component of the individual stocks.

$$\sigma_P^2 = \frac{1}{2} \sigma_e^2$$

- iii. Now consider four stocks, $i=\{1,2,3,4\}$. Construct a portfolio out of the four stocks that has zero exposure to the Size factor, an exposure of 1 to the Value factor, and exposure of 1.5 to the Momentum factor. Provide the system of equations to be used to solve for the weights. You do not need to find the explicit solution for the weights.

The weights solve the following system of equations:

$$w_1 + w_2 + w_3 + w_4 = 1$$

$$\gamma_1 w_1 + \gamma_2 w_2 + \gamma_3 w_3 + \gamma_4 w_4 = 0$$

$$\delta_1 w_1 + \delta_2 w_2 + \delta_3 w_3 + \delta_4 w_4 = 1$$

$$\kappa_1 w_1 + \kappa_2 w_2 + \kappa_3 w_3 + \kappa_4 w_4 = 1.5$$

NOTE: On the exam, there was no subscript on kappa. Hence, I accept solutions where you have solved with the same kappa for all 4 stocks!

Question 2: Portfolio Construction and Performance Measurement (15 points)

Part a. (2 points)

Theoretically, the standard deviation of a portfolio consisting of two equities can be reduced to what level? Explain. Realistically, is it possible to reduce the standard deviation to this level? Explain.

Theoretically, if one could find two securities with perfectly negatively correlated returns (correlation coefficient = -1), one could solve for the weights of these securities that would produce the minimum variance portfolio of these two securities. The standard deviation of the resulting portfolio would be equal to zero. However, in reality, securities with perfect negative correlations do not exist.

Part b. (3 points)

There are three stocks, A, B, and C. You can either invest in these stocks or short sell them. There are three possible states of nature for economic growth in the upcoming year; economic growth may be strong, moderate, or weak. The returns for the upcoming year on stocks A, B, and C for each of these states of nature are given below:

	State of Nature		
Stock	Strong Growth	Moderate Growth	Weak Growth
A	39%	17%	-5%
B	30%	15%	0%
C	6%	14%	22%

- i. If you invested in an equally weighted portfolio of stocks A and B, what would be your portfolio return if economic growth were moderate?

$$E(R_p) = 0.5(17\%) + 0.5(15\%) = 16\%.$$

- ii. How can you construct a risk-free arbitrage opportunity, assuming there is a 1/3 chance to be in each state of the nature? Explain and provide an example!

Short in C, Long in A and B. The return of a short position in C is always lower than a long in B and/or C. Below is an example (there are others, of course!)

$$E(R_A) = (39\% + 17\% - 5\%)/3 = 17\%; E(R_B) = (30\% + 15\% + 0\%)/3 = 15\%; E(R_C) = (22\% + 14\% + 6\%)/3 = 14\%; E(R_p) = -0.5(14\%) + 0.5[(17\% + 15\%)/2]; -7.0\% + 8.0\% = 1.0\%.$$

There have been more solutions suggested by you. I have examined each one individually and awarded it full points if correct.

Part c. (5 points)

Consider the following information about the performance of a fund and the market portfolio:

	Fund	Market portfolio
Average return	12%	9%
Standard deviation of returns	25%	20%
Beta	1.1	1

The risk free rate is 2%.

- i. Calculate the Sharpe ratio of the fund and of the market portfolio.

Sharpe ratio (fund) = $(0.12 - 0.02)/0.25 = 0.4$; Sharpe ratio (market) = $(0.09 - 0.02)/0.2 = 0.35$

- ii. Define the M2 performance measure, also giving a graphic interpretation. Hint: recall that it involves creating a hypothetical portfolio made up of a combination of the fund portfolio and a risk-free investment. What is the link between the M2 measure and the Sharpe ratio?

The M2 measure gives the excess return of a hypothetical portfolio over the market. The hypothetical portfolio is a linear combination of the risk-free asset and the managed portfolio, and has the same volatility as the market portfolio.

NOTE: Many of you had 0.5 points lower because they did not provide a graphical interpretation (Please see the slides and the book)

- iii. Calculate the weights of the hypothetical portfolio used to calculate the M2 measure. Obtain its return.

Weight(risk-free) = $1 - 0.2/0.25 = 0.2$; weight(managed portfolio) = $1 - 0.2 = 0.8$.

Return(hypothetic portfolio) = $0.8 \cdot 0.12 + 0.2 \cdot 0.02 = 0.1$

- iv. Calculate the M2 measure for the fund.

M2 = $0.1 - 0.09 = 0.01$

NOTE: Many of you gave the return of the hypothetical portfolio as the M2 measure (i.e. 0.1 instead of 0.01). I have deduced points for that, but still given points for giving the correct return of the hypothetical portfolio

Part d. (5 points)

You have a time series of realized returns of a mutual fund. According to the fund's prospectus, it invests in growth stocks and fixed income securities and has a stellar performance. You, however, suspect that the superior performance of this fund is due to investment in riskier value stocks. How can you investigate your concern, using style analysis? Explain the statistical tests you would perform!

You can run the following two models:

$$R_t = \alpha_1 + \beta_1 \text{Growth_stocks} + \gamma_1 \text{Bonds} \quad (1)$$

$$R_t = \alpha_2 + \beta_2 \text{Growth_stocks} + \gamma_2 \text{Bonds} + \delta_2 \text{Value_stocks} \quad (2)$$

Where R_t is the excess return of the fund, alpha indicate abnormal return of the fund, and *Growth_stocks*, *Bonds*, and *Value_stocks* stand for indices for growth stocks, fixed income securities, and value stocks, respectively. The beta and gamma coefficients in model 1 should sum up to one in order to represent the weights in the two styles. Similarly, the beta, gamma, and delta coefficients in model 2 should sum up to one in order to represent the weights in the three styles. The factor exposures give the return due to a style. The factor loadings can be estimated by minimizing the sum of squared residuals.

According to the prospectus of the fund, α_1 should be statistically different from zero. If indeed this is due to the investment in the riskier value stocks, then you should find a zero alpha in model 2 and a (significantly) positive δ_2 . You should also investigate whether $\alpha_2 < \alpha_1$ by means of a t-test.

NOTE: Some students have been creative in providing some ideas, other than style analysis. I have provided points for that!

Question 3: Fixed Income (15 points)

Part a. (3 points)

Discuss the meaning, usage, similarities and differences between the following 2 couples of concepts:

- i. duration of a bond and delta of an option
Duration measures the interest rate sensitivity of the bond price. The option delta measures the stock price sensitivity of the option. Both are linear approximations. The delta of the option gives the fractions of stocks needed to build a hedge portfolio; while this is not the case with duration (explicit duration matching is needed).
- ii. convexity of a bond and gamma of an option
Convexity measures the second order effect of interest rate changes on the bond price, while the gamma measures the second order effect of stock price changes on the option price.

Part b. (5 points)

Consider the following forward rates: $f_1 = 2\%$, $f_2 = 3\%$, and $f_3 = 4\%$.

- i. Express the prices of zero-coupon bonds with maturities of 1, 2, and 3 years using the forward rates. Solve for them.
 $P_1 = 100/(1+f_1) = 98.04$, $P_2 = 100/((1+f_1)*(1+f_2)) = 95.18$, $P_3 = 100/((1+f_1)*(1+f_2)*(1+f_3)) = 91.52$.
NOTE: I have used a face value of 100 above. Of course, I have accepted solutions with face values of 1000 and given full points.
- ii. Find the yield curve from the forward curve and explain how you obtained it.
 $z_1 = f_1$
 $(1+z_2)^2 = (1+f_1)*(1+f_2)$
 $(1+z_3)^3 = (1+f_1)*(1+f_2)*(1+f_3)$
Solving this system yields: $z_1 = 0.02$, $z_2 = 0.025$, $z_3 = 0.030$.
- iii. You are offered to enter in the following agreement: lend \$10000 at the start of year 3 at 5% for 1 year. What is the value of the agreement today? How do you found it?
 $-10000/(1+z_2)^2 + 10000*(1+0.05)/(1+z_3)^2 = \91.52 .
- iv. What are the cash-flows of the agreement (now (beginning of year 1), at the end of year 1, 2 and 3)? How can you hedge this agreement today? (Hint: use zero-coupon bonds). Give the portfolio for the hedging strategy and its cash-flows.
Cash-flows of the agreement:

now	91.52
1	0
2	-10000
3	10500

Hedging the agreement today:

now	Long 100 2-year zero bonds, short 105 3-year zero bonds, value of the position: $100 \cdot 98.04 - 105 \cdot 95.18$ = -91.52	
1	0	
2	-100*100	
3	105*100	

Note: If you did not solve correctly (iii) but solved correctly for (iv) using the incorrectly obtained value in (iii), you still receive full credit for part (iv)

Part c. (5 points)

You have a callable bond position. Its modified duration is 13 and convexity is 110. Also, you have 3 zero coupon bonds with maturities of 3, 5, and 15 years. The face value of the zero coupon bond is 100 euro, and the yield curve is flat at 4.5%.

- i. Calculate the prices, modified durations, and convexities of the three zero coupon bonds
The prices are calculate as $100/(1+\text{yield})^T$; durations equal maturities, modified durations are calculated as $\text{duration}/(1+\text{yield})$. The convexity of a zero coupon bond is $C = 1/(P \cdot (1+y)^2) \cdot 100 \cdot (T^2+T)/(1+y)^T = 1/(P \cdot (1+y)^2) \cdot P \cdot (T^2+T) = (T^2+T)/(1+y)^2$, where T is maturity, P is the price of the bond and y is the yield.

Zero Coupon	Price	Duration	Modified Duration	Convexity
3	87.63	3.00	2.87	10.99
5	80.25	5.00	4.78	27.47
15	51.67	15.00	14.35	219.78

- ii. Construct a portfolio of the three zero coupon bonds to hedge the callable bond position by matching its modified duration and taking a 10% position in the 3 year zero coupon bond. Obtain the weights on the other two bonds and calculate them.
Fixing the weight on bond one to 0.1, we need to solve for
 $2.87 \cdot w_1 + 4.78 \cdot w_2 + 14.35 \cdot w_3 = 13$
 $w_1 + w_2 + w_3 = 1$
With two unknowns and two equations, we solve $w_2 = 0.0215$ and $w_3 = 0.8785$

NOTE: If you did not solve correctly (i) but solved correctly for (ii) using the incorrectly obtained value in (i), you still receive full credit for part (ii)

- iii. Suggest a way to select the best combination of zero coupon bonds to hedge the callable bond exposure. Do not provide a solution, but state the equation (or system of equations).

The best solution would be obtained by matching on duration as well as convexity. Thus

$$w_1 + w_2 + w_3 = 1$$

$$2.87 * w_1 + 4.78 * w_2 + 14.35 * w_3 = 13$$

$$10.99 * w_1 + 27.47 * w_2 + 219.78 * w_3 = 110$$

NOTE: If you did not solve correctly (i) but solved correctly for (iii) using the incorrectly obtained value in (i) (i.e. you used the correct algorithm), you still receive full credit for part (iii)

Part d. (2 points)

Explain what the following terms mean: spot rate, short rate, and forward rate. Which of these is (are) observable today?

The n-period spot rate is the yield to maturity on a zero-coupon bond with a maturity of n periods. The short rate for period n is the one-period interest rate that will prevail in period n. The forward rate for period n is the short rate that would satisfy a "break-even condition" equating the total returns on two n-period investment strategies. The first strategy is an investment in an n-period zero-coupon bond. The second is an investment in an n-1 period zero-coupon bond "rolled over" into an investment in a one-period zero. Spot rates and forward rates are observable today, but because interest rates evolve with uncertainty, future short rates are not.

Question 4: Option Pricing (15 points)

Part a. (2 points)

Discuss marking to market and margin accounts in the futures market.

When opening an account, the trader establishes a margin account. The margin deposit may be cash or near cash, such as T-bills. Both sides of the contract must post margin. The initial margin is between 5 and 15% of the total value of the contract. The more volatile the asset, the higher the margin requirement. The clearinghouse recognizes profits and losses at the end of each trading day; this daily settlement is marking to market, thus proceeds accrue to the trader's account immediately; maturity date does not govern the realization of profits or losses.

Part b. (3 points)

Suppose you purchase 100 European call options on IBM maturing in May with an exercise price of 100 (premium=\$5) and write 100 call options on IBM maturing in May with an exercise price of 105 (the premium is \$2).

- i. What is the maximum potential profit of your strategy if both options are exercised?
 $-\$100 - \$5 = -\$105$; $+\$2 + \$105 = \$107$; $\$2 \times 100 = \200 .
- ii. How much would be your profit if, at expiration, the price of a share of IBM stock is \$103?
 $\$103 - \$100 = \$3$; $-(\$5 - \$2) = 0$; $\$0 \times 100 = \0 .
- iii. What is the maximum loss that you could suffer from your strategy?
 $-\$5 + \$2 = -\$3$; $-\$3 \times 100 = -\300 .
- iv. What is the lowest stock price at which you can break even?
 $x = \$100 + \$5 - \$2$; $x = \$103$.

Part c. (2 points)

An American-style call option with six months to maturity has a strike price of \$35. The underlying stock now sells for \$43. The call premium is \$12.

- i. What is the intrinsic value of the call?
 $43 - 35 = \$8$.
- ii. What is the time value of the call?

$$12 - (43 - 35) = \$4.$$

Part d. (4 points)

Consider an Asian call option. It pays the maximum between zero and the difference between the average realized price of the underlying during the life of the option (denote it by M), and the strike price (K). The price of the stock today is $S_0 = \$100$. Its future evolution can be modeled by a binomial tree, where at each period it can go up by 15% or down by 10%. The strike price is $K = \$93$. The periodic risk-free rate is 0.2%. The stock pays no dividends and the option can be exercised only at maturity.

Build a 2-period binomial tree (so for $t=0, t=1, t=2$) and calculate the price of the option based on it.

Price=8.10, using risk-neutral probabilities. See below for details.

$$Q = (S_0 * (1+rf) - S_d) / (S_u - S_d) = 0.408$$

Note below: The value of the stock in S_{ud} is the same as in S_{du} , but not the value of the call!

	1	2	3
			132.25
		115	103.5
Tree	100		103.5
		90	81
			22.75
		17.04	13.17
Value of Call	8.10		4.83
		1.97	0.00

Part e. (4 points)

Which of the variables affecting option pricing is not directly observable? How is it usually calculated in order to be used by, say, the Black-Scholes formula? If this variable is estimated to be higher or lower (using option pricing) than the variable actually is, then how is the option valuation affected?

The volatility of the underlying stock is not directly observable, but can be estimated from historic data. If the implied volatility is lower than the actual volatility of the stock, the option will be undervalued, as the higher the implied volatility, the higher the price of the option. Investors often use the implied volatility of the stock, i.e., the volatility of the stock implied by the price of the option. If investors think the actual volatility of the stock exceeds the implied volatility, the option would be considered to be underpriced. If actual volatility appears to be higher than the implied volatility, the "fair price" of the option would exceed the actual price.