

Exam: Investments 3.4

Code: E\_EBE3\_INV

Coordinator: dr. D. Stefanova

Date: March 25, 2013

Time: 8.45

Duration: 2 hours and 45 minutes

Calculator allowed: Yes

Graphical calculator allowed: Yes

Number of questions: 20 multiple choice questions and 4 open questions

Type of questions: Open/ multiple choice

Answer in: English

Remarks: Be concise and complete in your answers (including calculations). Always explain your answers, even if not explicitly called for. Use your time efficiently, using the maximum number of points per question as a guideline.

Credit score: The maximum possible scores for each part and question are indicated. In total, you can earn 100 points. Your final exam grade is determined by dividing the number of points by 10.

Grades: The grades will be made public on: April 8 2013.

Inspection: Monday, April 8 2013 at 10.00 in room 2A-33.

Number of pages: 10 (including front page)

**Good luck!**

**PART 1 (MULTIPLE CHOICE; 40 points at maximum)**

**Read the questions and answers carefully and write down your answer on your answer sheet. Your final score is determined as (# correct answers - 2) \* 40/18. Negative scores for this part of the exam are set to zero.**

1. Which of the following statements is (are) true?

- I) Risk-averse investors reject investments that are fair games.
- II) Risk-neutral investors judge risky investments only by the expected returns.
- III) Risk-averse investors judge investments only by their riskiness.
- IV) Risk-loving investors will not engage in fair games.

- A. I only
- B. II only
- C. I and II only
- D. II and III only
- E. II, III, and IV only

Risk-averse investors consider a risky investment only if the investment offers a risk premium. Risk-neutral investors look only at expected returns when making an investment decision.

2. An investor who wishes to form a portfolio that lies to the right of the optimal risky portfolio on the Capital Allocation Line must:

- A. lend some of her money at the risk-free rate and invest the remainder in the optimal risky portfolio.
- B. borrow some money at the risk-free rate and invest in the optimal risky portfolio.
- C. invest only in risky securities.
- D. such a portfolio cannot be formed.
- E. B and C

The only way that an investor can create portfolios to the right of the Capital Allocation Line is to create a borrowing portfolio (buy stocks on margin). In this case, the investor will not hold any of the risk-free security, but will hold only risky securities.

3. Given an optimal risky portfolio with expected return of 14% and standard deviation of 22% and a risk free rate of 6%, what is the slope of the best feasible CAL?

- A. 0.64
- B. 0.14
- C. 0.08
- D. 0.33
- E. 0.36

Slope =  $(14 - 6)/22 = .3636$

4. What is the expected return of a zero-beta security?

- A. The market rate of return.
- B. Zero rate of return.
- C. A negative rate of return.
- D. The risk-free rate.**
- E. None of the above.

$$E(R_S) = r_f + 0(R_M - r_f) = r_f.$$

5. The capital asset pricing model assumes

- A. all investors are price takers.
- B. all investors have the same holding period.
- C. investors pay taxes on capital gains.
- D. both A and B are true.**
- E. A, B and C are all true.

The CAPM assumes that investors are price-takers with the same single holding period and that there are no taxes or transaction costs.

6. Your opinion is that security A has an expected rate of return of 0.145. It has a beta of 1.5. The risk-free rate is 0.04 and the market expected rate of return is 0.11. According to the Capital Asset Pricing Model, this security is

- A. underpriced.
- B. overpriced.
- C. fairly priced.**
- D. cannot be determined from data provided.
- E. none of the above.

$$14.5\% = 4\% + 1.5(11\% - 4\%) = 14.5\%; \text{ therefore, the security is fairly priced.}$$

7. Low Fly Airline is expected to pay a dividend of \$7 in the coming year. Dividends are expected to grow at the rate of 15% per year. The risk-free rate of return is 6% and the expected return on the market portfolio is 14%. The stock of low Fly Airline has a beta of 3.00. The intrinsic value of the stock is \_\_\_\_\_.

- A. \$46.67**
- B. \$50.00
- C. \$56.00
- D. \$62.50
- E. none of the above

$$6\% + 3(14\% - 6\%) = 30\%; P = 7 / (.30 - .15) = \$46.67.$$

8. Suppose two portfolios have the same average return, the same standard deviation of returns, but Buckeye Fund has a higher beta than Gator Fund. According to the Treynor measure, the performance of Buckeye Fund

- A. is better than the performance of Gator Fund.
- B. is the same as the performance of Gator Fund.
- C. is poorer than the performance of Gator Fund.**
- D. cannot be measured as there is no data on the alpha of the portfolio
- E. none of the above is true.

The Treynor index is a measure of average portfolio returns (in excess of the risk free return) per unit of systematic risk (as measured by beta).

9. In the APT model, what is the nonsystematic standard deviation of an equally-weighted portfolio that has an average value of  $\sigma(e_i)$  equal to 25% and 50 securities?

- A. 12.5%
- B. 625%
- C. 0.5%
- D. 3.54%**
- E. 14.59%

$$\sigma^2(e_p) = \frac{1}{n} \sigma^2(e_i) = \frac{1}{50} (25)^2 = 12.5, \quad \sigma(e_p) = \sqrt{12.5} = 3.54\%$$

10. According to the Capital Asset Pricing Model (CAPM), fairly priced securities

- A. have positive betas.
- B. have zero alphas.**
- C. have negative betas.
- D. have positive alphas.
- E. none of the above.

A zero alpha results when the security is in equilibrium (fairly priced for the level of risk).

11. Consider two bonds, A and B. Both bonds presently are selling at their par value of \$1,000. Each pays interest of \$120 annually. Bond A will mature in 5 years while bond B will mature in 6 years. If the yields to maturity on the two bonds change from 12% to 10%, \_\_\_\_\_.

- A. both bonds will increase in value, but bond A will increase more than bond B
- B. both bonds will increase in value, but bond B will increase more than bond A**
- C. both bonds will decrease in value, but bond A will decrease more than bond B
- D. both bonds will decrease in value, but bond B will decrease more than bond A
- E. none of the above

The longer the maturity, the greater the price change when interest rates change.

12. When a bond indenture includes a sinking fund provision

- A. firms must establish a cash fund for future bond redemption.
- B. bondholders always benefit, because principal repayment on the scheduled maturity date is guaranteed.
- C. bondholders may lose because their bonds can be repurchased by the corporation at below-market prices.
- D. both A and B are true.
- E. none of the above are true.

A sinking fund provisions requires the firm to redeem bonds over several years, either by open market purchase or at a special call price from bondholders. This can result in repurchase in advance of scheduled maturity at below-market prices.

13. What is the relationship between the price of a straight bond and the price of a callable bond?

- A. The straight bond's price will be higher than the callable bond's price for low interest rates.
- B. The straight bond's price will be lower than the callable bond's price for low interest rates.
- C. The straight bond's price will change as interest rates change, but the callable bond's price will stay the same.
- D. The straight bond and the callable bond will have the same price.
- E. There is no consistent relationship between the two types of bonds.

For low interest rates, the price difference is due to the value of the firm's option to call the bond at the call price. The firm is more likely to call the issue at low interest rates, so the option is valuable. At higher interest rates the firm is less likely to call and this option loses value. The prices converge for high interest rates.

14. The pure yield curve can be estimated

- a. by using zero-coupon bonds.
- b. by using coupon bonds if each coupon is treated as a separate "zero."
- c. by using corporate bonds with different risk ratings.
- d. by estimating liquidity premiums for different maturities.
- E. A and B.

The pure yield curve is calculated using zero coupon bonds, but coupon bonds may be used if each coupon is treated as a separate "zero."

15. Some of the problems with immunization are

- A. duration assumes that the yield curve is flat.
- B. duration assumes that if shifts in the yield curve occur, these shifts are parallel.
- C. immunization is valid for one interest rate change only.
- D. durations and horizon dates change by the same amounts with the passage of time.
- E. A, B, and C.

Durations and horizon dates change with the passage of time, but not by the same amounts.

16. Which of the following factors affect the price of a stock option

- A. the risk-free rate.
- B. the riskiness of the stock.
- C. the time to expiration.
- D. the expected rate of return on the stock.
- E. A, B, and C.

A, B, and C are directly related to the price of the option; D does not affect the price of the option.

17. All the inputs in the Black-Scholes Option Pricing Model are directly observable **except**

- A. the price of the underlying security.
- B. the risk free rate of interest.
- C. the time to expiration.
- D. the variance of returns of the underlying asset return.
- E. none of the above.

The variance of the returns of the underlying asset is not directly observable, but must be estimated from historical data, from scenario analysis, or from the prices of other options.

18. Delta neutral

- A. is the volatility level for the stock that the option price implies.
- B. is the continued updating of the hedge ratio as time passes.
- C. is the percentage change in the stock call option price divided by the percentage change in the stock price.
- D. means the portfolio has no tendency to change value as the underlying portfolio value changes.
- E. A and C.

Delta neutral means the portfolio has no tendency to change value as the underlying portfolio value changes.

19. The Black-Scholes formula assumes that

- I) the risk-free interest rate is constant over the life of the option.
- II) the stock price volatility is constant over the life of the option.
- III) the expected rate of return on the stock is constant over the life of the option.
- IV) there will be no sudden extreme jumps in stock prices.

- A. I and II
- B. I and III
- C. II and II
- D.** I, II and IV
- E. I, II, III, and IV

The risk-free rate and stock price volatility are assumed to be constant but the option value does not depend on the expected rate of return on the stock. The model also assumes that stock prices will not jump markedly.

20. A covered call position is

- A. the simultaneous purchase of the call and the underlying asset.
- B. the purchase of a share of stock with a simultaneous sale of a put on that stock.
- C. the short sale of a share of stock with a simultaneous sale of a call on that stock.
- D.** the purchase of a share of stock with a simultaneous sale of a call on that stock.
- E. the simultaneous purchase of a call and sale of a put on the same stock.

Writing a covered call is a very safe strategy, as the writer owns the underlying stock. The only risk to the writer is that the stock will be called away, thus limiting the upside potential.

**PART 2 (OPEN QUESTIONS; 6 0 points at maximum)**

**QUESTION 1. (15 points) Equilibrium pricing models**

**Part a. (4 points)**

What are the equilibrium implications of the CAPM? When answering this question focus on the following:

- i. Give a graphical illustration of the Capital Market Line and explain it.
- ii. State the mutual fund theorem.
- iii. Give an expression for the risk premium of the market portfolio
- iv. Given an expression for the risk premium of individual assets (the expected return-beta relationship)

All investors choose to hold the same (market) portfolio, which is the tangency portfolio to the efficient frontier: the one with the best attainable CAL. The allocation between the market portfolio and the risk-free rate depends on individual investor's risk aversion. The risk premium of the market portfolio is given by:

$$E[r_M] - r_f = \bar{A}\sigma_M^2$$

where  $E[r_M]$  is the expected market return,  $r_f$  is the risk-free rate,  $\bar{A}$  is the degree of risk aversion of the representative investor, and  $\sigma_M^2$  is the variance of the market portfolio. The risk premium of individual assets is given by:

$$E[r_i] - r_f = \beta_i(E[r_M] - r_f)$$

where  $\beta_i = \frac{\text{cov}(r_M, r_i)}{\sigma_M^2}$ .

**Part b. (3 points)**

Assume that the CAPM holds. The risk-free rate  $r_f$  is 2% and the market return  $r_M$  is 7%. Consider the following three stocks:

Stocks	$\beta$	ROE	b
A	1.2	0.1	0.1
B	1	0.1	0.2
C	0.8	0.1	0.4

$\beta$  - market beta of each stock

ROE – return on equity

b – retention ratio

What is the required rate of return of the three stocks? Compute the P/E ratio of the three stocks, assuming a constant growth model with endogenous earnings growth. Why do the three P/E ratios differ?

The P/E ratio is computed using the formula:  $P/E = (1-b)/(k - b \times \text{ROE})$ . The required rate of return  $k$  for the three stocks:



$k_A = 0.02 + 1.2 \times (0.07 - 0.02) = 0.08$ ;  $k_B = 0.02 + 1 \times (0.07 - 0.02) = 0.07$ ;  $k_C = 0.02 + 0.8 \times (0.07 - 0.02) = 0.06$ .

$P/E_A = 0.9/(0.08 - (1-0.9) \times 0.1) = 12.86$ ;  $P/E_B = 0.8/(0.07 - (1-0.8) \times 0.1) = 16$ ;  $P/E_C = 0.6/(0.06 - (1-0.6) \times 0.1) = 30$ ;

$ROE > k$  for each of the stocks; in this case the higher the retention ratio, the higher the P/E ratio.

### Part c. (5 points)

Consider the multifactor APT. There are two independent economic factors,  $F_1$  and  $F_2$ . The risk-free rate of return is  $r_f$ . You have the following information on two well-diversified portfolios:

Portfolio	$\beta$ on $F_1$	$\beta$ on $F_2$	Expected return
A	0.5	0	$r_A$
B	1	0.5	$r_B$

- Assuming that no arbitrage opportunities exist, give an expression for the risk premiums of the two factors.
- If the variances of the two factors are  $\sigma^2(F_1)$  and  $\sigma^2(F_2)$ , give an expression for the systematic risk of each one of the two portfolios.
- Construct a portfolio consisting of A and B which has zero exposure to  $F_1$  risk. What would the weights in A and B be?

- (i) Denoting the risk premium of factor  $F_1$  by  $RP(F_1)$  and the risk premium of factor  $F_2$  by  $RP(F_2)$ , we have that:

$$r_A = r_f + 0.5RP(F_1), \text{ thus } RP(F_1) = 2(r_A - r_f)$$

$$r_B = r_f + RP(F_1) + 0.5RP(F_2), \text{ thus } RP(F_2) = (r_B - r_f - RP(F_1)) / 0.5 = 2(r_B - r_f - 2(r_A - r_f))$$

- (ii)  $\sigma_A^2 = 0.5^2 \times \sigma^2(F_1)$  and  $\sigma_B^2 = \sigma^2(F_1) + 0.5 \times \sigma^2(F_2)$

- (iii) The weights of the portfolio with exposure of 1 to  $F_1$  solve the following system of equations:

$$w_A + w_B = 1$$

$$0.5w_A + w_B = 0$$

Thus for the weights we obtain:  $0.5w_A + 1 - w_A = 0$ ,  $w_A = 2$ , hence the weight in B is  $w_B = -1$

### Part d. (3 points)

List at least two testable implications of the CAPM. Write the regression equations that you would use in order to test them. Given that, what parameter restrictions would you test for?

Run  $n$  first-pass time series regressions of the excess return of the  $n$  risky stocks on the market excess return  $r_i - r_f = \alpha_i + b_i(r_M - r_f)$ . Then run a cross-sectional second-pass regression

$$\text{Avg}(r_i - r_f) = g_0 + g_1 b_i + e_{it}$$

where:  $\text{Avg}(r_i - r_f)$  = the average difference over the time series between the return on stock  $i$  and the risk-free rate,  $b_i$  = the beta of stock  $i$  from the first pass regression.

If CAPM is valid, then  $g_0 = 0$  and  $g_1$  equals to the average difference between the return on the market portfolio and the risk-free rate (i.e. the market risk premium).

## QUESTION 2. (15 points) Portfolio construction and performance measurement

### Part a. (6 points)

Consider two perfectly negatively correlated risky securities A and B. A has an expected rate of return of 12% and a standard deviation of 17%. B has an expected rate of return of 9% and a standard deviation of 14%.

- i. If you form a portfolio with weights  $w_A$  in the security A and  $w_B$  in the security B, what would the portfolio variance be? Express it as a function of the weights in both securities.
  - ii. If your goal is to hold a portfolio with the lowest possible variance, what proportions would you hold securities A and B in? What is the variance of the portfolio in that case?
  - iii. Construct a risk-free portfolio using securities A and B only. Calculate its rate of return.
- (i) Since both securities are perfectly negatively correlated, the variance of the portfolio is given by  $\sigma_P^2 = (0.17w_A - 0.14w_B)^2$
- (ii)  $w_A = 14/(17 + 14) = 0.45$ ;  $w_B = 1 - 0.45 = 0.55$ . The variance of the resulting portfolio is zero.
- (iii) The portfolio in (ii) is a perfectly hedged risk-free portfolio (zero variance). Its return is  $0.45(12\%) + 0.55(9\%) = 10.35\%$ .

### Part b. (5 points)

Consider an asset P with expected rate of return of  $E[r_P] = 15\%$  and a standard deviation  $\sigma_P$  of 20%. Further, consider an investor who has the following utility function:

$$U = E[r_{CP}] - \frac{1}{2} A \sigma_{CP}^2$$

where  $A=5$  is the level of risk aversion of the investor,  $E[r_{CP}]$  is the expected return of his portfolio, and  $\sigma_{CP}^2$  is its variance. As well, the risk-free return is 5%.

- i. If you invest  $w$  in asset P and  $(1-w)$  in the risk-free asset, what is the expected rate of return of your portfolio (as a function of  $w$ )? And its standard deviation?
  - ii. Obtain the weight  $w^*$  in asset P that maximizes the investor's utility function, if he forms a portfolio of P and the risk-free asset.
  - iii. Which value of  $A$  makes the investor indifferent between investing in the risky asset P and the risk-free asset?
- (i)  $E[r_{CP}] = w(15\%) + (1 - w)(5\%)$  and  $\sigma_{CP} = w(20\%)$
- (ii)  $w^* = \frac{0.15 - 0.05}{5 \cdot 0.2^2}$
- (iii)  $U(r_P) = U(r_f)$ , thus  $E[r_P] - \frac{1}{2} A \sigma_P^2 = r_f$  or  $A = \frac{2(E[r_P] - r_f)}{\sigma_P^2} = \frac{2(0.15 - 0.05)}{0.2^2}$

**Part c. (4 points)**

The following data are available relating to the performance of Sooner Stock Fund and the market portfolio:

	Sooner	Market Portfolio
Average Return	20%	11%
Standard Deviation of Returns	44%	19%
Beta	1.8	1.0
Residual standard deviation	2.0%	0.0%

The risk-free return during the sample period was 3%.

- i. What is the Sharpe measure of performance evaluation for Sooner Stock Fund?
- ii. Calculate the Jensen measure of performance evaluation for Sooner Stock Fund.
- iii. Calculate the information ratio for Sooner Stock Fund.

(i)  $(20\% - 3\%) / 44\% = 0.386$ , or 38.6%.

(ii)  $\alpha_p = 20\% - [3\% + 1.8(11\% - 3\%)] = 2.6\%$ .

(iii)  $2.6\% / 2.00\% = 1.3$ .

**QUESTION 3. (15 points) Fixed Income**
**Part a. (6 points)**

The yield to maturity on 1-year zero-coupon bonds is currently 7%; the YTM on 2-year zeros is 8%. The Treasury plans to issue a 2-year maturity *coupon* bond, paying coupons once per year with a coupon rate of 10%. The face value of the bond is \$100.

- i. At what price will the bond sell?
- ii. What will the yield to maturity on the bond be? (write the equation that should be used to obtain it)
- iii. If the expected short rate one year from now equals the forward rate next year (i.e. the expectations theory of the yield curve is correct), what is the market expectation of the price that the bond will sell for next year?
- iv. Recalculate your answer to (c) if you believe in the liquidity preference theory and you believe that the liquidity premium is 2%.

(i)  $P = \$10/1.07 + \$110/1.08^2 = \$103.65$

(ii) The yield to maturity is the solution for  $y$  in the following equation:  $\$10/(1+y) + \$110/(1+y)^2 = \$103.65$

Using a financial calculator, enter  $n = 2$ ;  $FV = 100$ ;  $PMT = 10$ ;  $PV = -103.65$ ; Compute  $i$ .  $YTM = 7.953\%$

- (iii) The forward rate for next year, derived from the zero-coupon yield curve, is the solution for  $f_2$  in the following equation:  $1+f_2 = 1.08^2/1.07 = 1.0901 \rightarrow f_2 = 9.01\%$   
Therefore, using an expected rate for next year of  $r_2 = 9.01\%$ , we find that the forecast bond price is  $P = \$110/1.0901 = \$100.91$
- (iv) If the liquidity premium is 2% then the forecast interest rate is:  $E(r_2) = f_2 - \text{liquidity premium} = 9.01\% - 2\% = 7.01\%$ . The forecast of the bond price is  $\$110/1.0701 = \$102.79$

**Part b. (2 points)**

A 9-year bond has a yield of 12.0% and a duration of 8.641 years. If the market yield increases by 75 basis points, what is the percentage change in the bond's price?

The percentage bond price change is:

$$-\frac{\text{Duration}}{1+y} \times \Delta y = -\frac{8.641}{1.120} \times .0075 = -.0579 = -5.79\% \text{ or a } 5.79\% \text{ decline}$$

**Part c. (5 points)**

You are managing a portfolio of \$1 million. Your target duration is 10 years, and you can choose from two bonds: a zero-coupon bond with maturity of 5 years, and a perpetuity, each currently yielding 5.3%.

- i. Calculate the duration of the zero-coupon bond and the perpetuity.
- ii. What weight of each bond will you hold to immunize your portfolio (i.e. achieve your target duration)?
- iii. How will these weights change *next year* if target duration is 9 years?

- (i) The duration of the perpetuity is:  $1.053/0.053 = 19.87$  years. The duration of the 5-year zero is 5 years.
- (ii) Call  $w$  the weight of the zero-coupon bond. Then:  $(w \times 5) + [(1 - w) \times 19.87] = 10 \rightarrow w = 9.87/14.87 = .6637 = 66.37$   
Therefore, the portfolio weights would be as follows: 66.37 invested in the zero and 33.63 in the perpetuity.
- (iii) Next year, the zero-coupon bond will have a duration of 4 years and the perpetuity will still have a 19.87-year duration. To obtain the target duration of nine years, which is now the duration of the obligation, we again solve for  $w$ :  
 $(w \times 4) + [(1 - w) \times 19.87] = 9 \rightarrow w = 10.8679/15.8679 = .6849 = 68.49$   
So, the proportion of the portfolio invested in the zero increases to 68.49 and the proportion invested in the perpetuity falls to 31.51.

**Part d. (2 points)**

The zero rates for maturities of 1, 2, and 3 years are given by  $z_1$ ,  $z_2$ , and  $z_3$ . Obtain (analytically) the corresponding one-year forward rates for  $t=1, 2, 3$  ( $f_1, f_2, f_3$ ) in terms of the zero rates. Express the price of a 3-year coupon bond with annual coupon payments of  $C$  and face value of 100 using the forward rates.

The forward rates from the zero curve:

$$\begin{aligned}f_1 &= z_1 \\f_2 &= \frac{(1 + z_2)^2}{1 + z_1} - 1 \\f_3 &= \frac{(1 + z_3)^3}{(1 + z_2)^2} - 1\end{aligned}$$

The price of a coupon bond and 3 years maturity using the forward rates:

$$P = \frac{C}{1 + f_1} + \frac{C}{(1 + f_1)(1 + f_2)} + \frac{C + 100}{(1 + f_1)(1 + f_2)(1 + f_3)}$$

<b>QUESTION 4. (15 points) Option pricing</b>
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**Part a. (3 points)**

Consider the following strategy. You buy a share of stock, write a 1-year call option on it with  $X = \$100$ , and buy a 1-year put option on the same stock with  $X = \$100$ . Your cost to establish the entire portfolio is \$99.1. The stock pays no dividends.

- i. What is the pay-off at year-end of that strategy?
- ii. Determine the risk-free rate.

The following payoff table shows that the portfolio is riskless with time-T value equal to \$100:

Position	$S_T < 100$	$S_T > 100$
Buy stock	$S_T$	$S_T$
Write call, X = \$100	0	$(S_T - 100)$
Buy put, X = \$100	$100 - S_T$	0
Total	100	100

Therefore, the risk-free rate is:  $(\$100/\$99.1) - 1 = 0.0091 = 0.91\%$

### Part b. (4 points)

You are holding call options on a stock. The stock's beta is 0.76, and you are concerned that the stock market is about to fall. The stock is currently selling for \$16 and you hold 1 million options on the stock (i.e., you hold 10,000 contracts for 100 shares each). The option delta is 0.80.

- If the stock market index decreases by 1%, by how much would the stock be expected to decrease? Give the dollar amount by which the 1 million shares of the stock would be expected to decrease.
- By how much would the value of your portfolio of options change in that case?
- How much of the market index portfolio must you buy or sell to hedge your market exposure?

If the stock market index decreases by 1%, the 1 million shares of stock on which the options are written would be expected to decrease by:

$$0.76\% \times \$16 \times 1 \text{ million} = \$121,600$$

The options would decrease by:

$$\text{delta} \times \$121,600 = .80 \times \$121,600 = \$97,280$$

In order to hedge your market exposure, you must sell \$9,728,000 of the market index portfolio so that a 1% change in the index would result in a \$97,280 change in the value of the portfolio.

### Part c. (8 points)

The price of a stock today (at  $t=0$ ) is \$100. The annual risk-free rate is 6%. Build a binomial tree for the evolution of the stock price over the period of 1 year assuming 2 steps ( $t=0$ ,  $t=1$ ,  $t=2$ ).

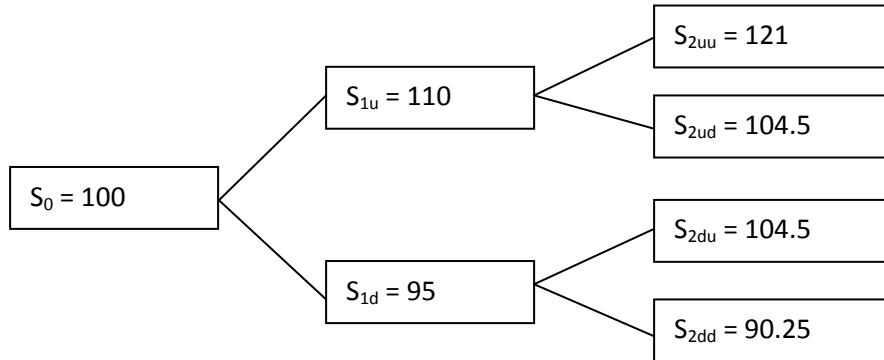
The stock price can increase by 10% and decrease by 5% each period.

- Obtain the price a put option on that stock along that tree. The strike price is \$110.
- Alternatively, obtain the price of an up-and-out barrier option. This is a path-dependent option that expires if the underlying price reaches a pre-determined knock-out barrier. This option behaves like a standard one if the underlying price remains below the barrier. If however it moves above the barrier, the contract

expires (regardless of whether the price moves back down afterwards or not). The knock-out barrier is \$105. The strike price is \$110.

- iii. Offer an explanation on the difference in the prices of both options.

The binomial tree:



The risk-neutral probability of an upward move is calculated using  $Q = (S_0(1+rf)-S_d)/(S_u-S_d) = (100(1+0.03)-95)/(110-95) = 0.53$

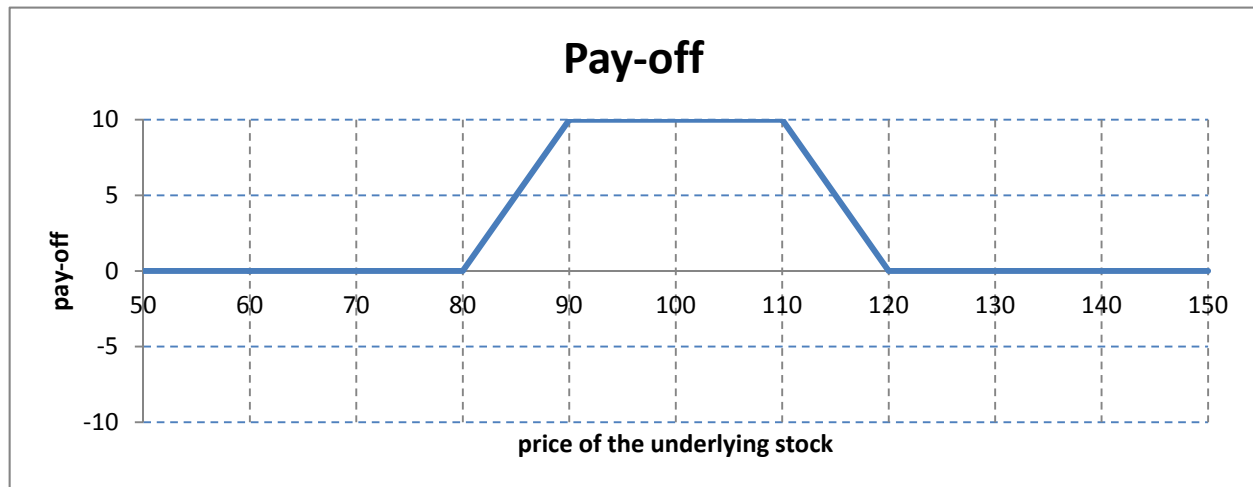
Up-and-out put: at the up node of step one of the tree the stock price  $S_u = \$110 > \$105$ , thus the option expires. Thus at the last node  $C_{uu} = C_{ud} = 0$ . For the other two nodes  $C_{du} = \max(0, 110 - 104.5) = 5.5$  and  $C_{dd} = \max(0, 110 - 90.25) = 19.75$ . At the first step:  $C_u = 0$  and  $C_d = (0.53 \times 5.5 + (1-0.53) \times 19.75)/(1+0.03) = 11.8$ . Thus, the price of the put is  $C_0 = (0.53 \times 0 + (1-0.53) \times 11.8)/(1+0.03) = 5.34$ .

Plain vanilla put:  $C_{uu} = 0$ ,  $C_{ud} = C_{du} = 5.5$ ,  $C_{dd} = 19.75$ . At step 1:  $C_u = (0.53 \times 0 + (1-0.53) \times 5.5)/(1+0.03) = 2.49$  and  $C_d = (0.53 \times 5.5 + (1-0.53) \times 19.75)/(1+0.03) = 11.8$ . At step 0 the price of the plain vanilla put is  $C_0 = (0.53 \times 2.49 + (1-0.53) \times 11.8)/(1+0.03) = 6.63$ .

#### Part d. (2 points)

Give an example of an options pay-off structure that allows you to earn a limited profit when the price of the underlying security has little volatility (small movements of the price in either direction), while you have a limited risk on the downside (limited losses for large movements of the price in either direction). Assume that you seek to achieve a maximum pay-off of \$10 and a minimum pay-off of \$0. Current stock price is \$100 and you can sell or buy in-, out- or at-the money options. Your options strategy should give you a non-negative pay-off if the price of the underlying moves between \$80 and \$120, and zero pay-off otherwise.

Draw a diagram of the desired pay-off structure. Determine the options positions that you will use to replicate it.



- 1 short ITM call @ 90
- 1 long ITM call @ 80
- 1 short OTM call @ 110
- 1 long OTM call @ 120