

Exam: Investments 3.4

Code: E\_EBE3\_INV

Coordinator: dr. D. Stefanova

Date: March 26, 2012

Time: 15.15

Duration: 2 hours and 45 minutes

Calculator allowed: Yes

Graphical calculator allowed: Yes

Number of questions: 20 multiple choice questions and 4 open questions

Type of questions: Open/ multiple choice

Answer in: English

Remarks: Be concise and complete in your answers (including calculations). Always explain your answers, even if not explicitly called for. Use your time efficiently, using the maximum number of points per question as a guideline.

Credit score: The maximum possible scores for each part and question are indicated. In total, you can earn 100 points. Your final exam grade is determined by dividing the number of points by 10.

Grades: The grades will be made public on: April 9 2010.

Inspection: Wednesday, April 11 2010 at 10.00 in room 2A-33.

Number of pages: 12 (including front page)

**Good luck!**

**PART 1 (MULTIPLE CHOICE; 40 points at maximum)**

**Read the questions and answers carefully and write down your answer on your answer sheet. Your final score is determined as (# correct answers - 2) \* 40/18. Negative scores for this part of the exam are set to zero.**

1. An investor invests 30 percent of his wealth in a risky asset with an expected rate of return of 0.15 and a variance of 0.04 and 70 percent in a T-bill that pays 6 percent. His portfolio's expected return and standard deviation are \_\_\_\_\_ and \_\_\_\_\_, respectively.

- A. 0.114; 0.12
- B. 0.087; 0.06**
- C. 0.295; 0.12
- D. 0.087; 0.12
- E. 0.795; 0.14

$E(r_p) = 0.3(15\%) + 0.7(6\%) = 8.7\%$ ;  $s_p = 0.3(0.04)^{1/2} = 6\%$ .

2. Consider an investment opportunity set formed with two securities that are perfectly negatively correlated. The global minimum variance portfolio has a standard deviation that is always

- A. greater than zero.
- B. equal to zero.**
- C. equal to the sum of the securities' standard deviations.
- D. equal to -1.
- E. between zero and -1.

If two securities were perfectly negatively correlated, the weights for the minimum variance portfolio for those securities could be calculated, and the standard deviation of the resulting portfolio would be zero.

3. In the single-index model represented by the equation  $r_i = E(r_i) + \beta_i F + e_i$ , the term  $e_i$  represents

- A. the impact of unanticipated macroeconomic events on security i's return.
- B. the impact of unanticipated firm-specific events on security i's return.**
- C. the impact of anticipated macroeconomic events on security i's return.
- D. the impact of anticipated firm-specific events on security i's return.
- E. the impact of changes in the market on security i's return.

The textbook discusses a model in which macroeconomic events are used as a single index for security returns. The  $e_i$  term represents the impact of unanticipated firm-specific events. The  $e_i$  term has an expected value of zero. Only unanticipated events would affect the return.

4. According to the Capital Asset Pricing Model (CAPM),
- A. a security with a positive alpha is considered overpriced.
  - B. a security with a zero alpha is considered to be a good buy.
  - C. a security with a negative alpha is considered to be a good buy.
  - D.** a security with a positive alpha is considered to be underpriced.
  - E. a security with a positive beta is considered to be underpriced.

A security with a positive alpha is one that is expected to yield an abnormal positive rate of return, based on the perceived risk of the security, and thus is underpriced.

5. A security has an expected rate of return of 0.10 and a beta of 1.1. The market expected rate of return is 0.08 and the risk-free rate is 0.05. The alpha of the stock is

- A.** 1.7%.
- B. -1.7%.
- C. 8.3%.
- D. 5.5%.
- E. -5.5%.

$$10\% - [5\% + 1.1(8\% - 5\%)] = 1.7\%.$$

6. Consider the multifactor model APT with two factors. Portfolio A has a beta of 0.75 on factor 1 and a beta of 1.25 on factor 2. The risk premiums on the factor 1 and factor 2 portfolios are 1% and 7%, respectively. The risk-free rate of return is 7%. The expected return on portfolio A is \_\_\_\_\_ if no arbitrage opportunities exist.

- A. 13.5%
- B. 15.0%
- C.** 16.5%
- D. 23.0%
- E. 18.7%

$$7\% + 0.75(1\%) + 1.25(7\%) = 16.5\%.$$

7. In an efficient market the correlation coefficient between stock returns for two non-overlapping time periods should be

- A. positive and large.
- B. positive and small.
- C.** zero.
- D. negative and small.
- E. negative and large.

In an efficient market there should be no serial correlation between returns from non-overlapping periods.

8. Each of two stocks, C and D, are expected to pay a dividend of \$3 in the upcoming year. The expected growth rate of dividends is 9% for both stocks. You require a rate of return of 10% on stock C and a return of 13% on stock D. The intrinsic value of stock C \_\_\_\_.

- A.** will be greater than the intrinsic value of stock D
- B. will be the same as the intrinsic value of stock D
- C. will be less than the intrinsic value of stock D
- D. cannot be calculated without knowing the market rate of return
- E. None of these is correct.

$PV_0 = D_1/(k-g)$ ; given that dividends are equal, the stock with the larger required return will have the lower value.

9. Suppose two portfolios have the same average return, the same standard deviation of returns, but Aggie Fund has a higher beta than Raider Fund. According to the Sharpe measure, the performance of Aggie Fund

- A. is better than the performance of Raider Fund.
- B.** is the same as the performance of Raider Fund.
- C. is poorer than the performance of Raider Fund.
- D. cannot be measured as there is no data on the alpha of the portfolio.
- E. None of these is correct.

The Sharpe index is a measure of average portfolio returns (in excess of the risk free return) per unit of total risk (as measured by standard deviation).

10. You want to evaluate three mutual funds using the Treynor measure for performance evaluation. The risk-free return during the sample period is 6%. The average returns, standard deviations, and betas for the three funds are given below, in addition to information regarding the S&P 500 index.

	Average Return	Standard. Deviation	Beta
Fund A	13%	10%	0.5
Fund B	19%	20%	1.0
Fund C	25%	30%	1.5
S&P 500	18%	16%	1.0

The fund with the highest Treynor measure is \_\_\_\_\_.

- A.** Fund A
- B. Fund B
- C. Fund C
- D. Funds A and B are tied for highest
- E. Funds A and C are tied for highest

A:  $(13\% - 6\%)/0.5 = 14$ ; B:  $(19\% - 6\%)/1.0 = 13$ ; C:  $(25\% - 6\%)/1.5 = 12.7$ ; S&P 500:  $(18\% - 6\%)/1.0 = 12$ .

11. A Treasury bond due in one year has a yield of 6.2%; a Treasury bond due in 5 years has a yield of 6.7%. A bond issued by Xerox due in 5 years has a yield of 7.9%; a bond issued by Exxon due in one year has a yield of 7.2%. The default risk premiums on the bonds issued by Exxon and Xerox, respectively, are

- A.** 1.0% and 1.2%
- B. 0.5% and .7%
- C. 1.2% and 1.0%
- D. 0.7% and 0.5%
- E. None of these is correct.

Exxon:  $7.2\% - 6.2\% = 1.0\%$ ; Xerox:  $7.9\% - 6.7\% = 1.2\%$ .

12. Consider two bonds, A and B. Both bonds presently are selling at their par value of \$1,000. Each pays interest of \$120 annually. Bond A will mature in 5 years while bond B will mature in 6 years. If the yields to maturity on the two bonds change from 12% to 10%, \_\_\_\_\_.

- A. both bonds will increase in value, but bond A will increase more than bond B
- B.** both bonds will increase in value, but bond B will increase more than bond A
- C. both bonds will decrease in value, but bond A will decrease more than bond B
- D. both bonds will decrease in value, but bond B will decrease more than bond A
- E. None of these is correct.

The longer the maturity, the greater the price change when interest rates change.

13. The **pure yield curve** can be estimated

- A. by using zero-coupon Treasuries.
- B. by using stripped Treasuries if each coupon is treated as a separate "zero."
- C. by using corporate bonds with different risk ratings.
- D. by estimating liquidity premiums for different maturities.
- E.** by using zero-coupon Treasuries and by using stripped Treasuries if each coupon is treated as a separate "zero."

The pure yield curve is calculated using stripped or zero coupon Treasuries.

14. Given the time to maturity, the duration of a zero-coupon bond is higher when the discount rate is

- A. higher.
- B. lower.
- C. equal to the risk free rate.
- D.** The bond's duration is independent of the discount rate.
- E. None of these is correct.

The duration of a zero-coupon bond is equal to the maturity of the bond.

15. Immunization is not a strictly passive strategy because

- A. it requires choosing an asset portfolio that matches an index.
- B. there is likely to be a gap between the values of assets and liabilities in most portfolios.
- C.** it requires frequent rebalancing as maturities and interest rates change.
- D. durations of assets and liabilities fall at the same rate.
- E. None of these is correct.

As time passes the durations of assets and liabilities fall at different rates, requiring portfolio rebalancing. Further, every change in interest rates creates changes in the durations of portfolio assets and liabilities.

16. The current market price of a share of AT&T stock is \$50. If a call option on this stock has a strike price of \$45, the call

- A. is out of the money.
- B. is in the money.
- C. sells for a higher price than if the market price of AT&T stock is \$40.
- D. is out of the money and sells for a higher price than if the market price of AT&T stock is \$40.
- E.** is in the money and sells for a higher price than if the market price of AT&T stock is \$40.

If the striking price on a call option is less than the market price, the option is in the money and sells for more than an out of the money option.

17. A covered call position is

- A. the simultaneous purchase of the call and the underlying asset.
- B. the purchase of a share of stock with a simultaneous sale of a put on that stock.
- C. the short sale of a share of stock with a simultaneous sale of a call on that stock.
- D. the purchase of a share of stock with a simultaneous sale of a call on that stock.
- E. the simultaneous purchase of a call and sale of a put on the same stock.

Writing a covered call is a very safe strategy, as the writer owns the underlying stock. The only risk to the writer is that the stock will be called away, thus limiting the upside potential.

18. Which of the following factors affect the price of a stock option

- A. the risk-free rate.
- B. the riskiness of the stock.
- C. the time to expiration.
- D. the expected rate of return on the stock.
- E. the risk-free rate, the riskiness of the stock, and the time to expiration.

The risk-free rate, the riskiness of the stock, and the time to expiration are directly related to the price of the option; the expected rate of return on the stock does not affect the price of the option.

19. Which of the inputs in the Black-Scholes Option Pricing Model are directly observable

- A. the price of the underlying security.
- B. the risk free rate of interest.
- C. the time to expiration.
- D. the variance of returns of the underlying asset return.
- E. the price of the underlying security, the risk free rate of interest, and the time to expiration.

The variance of the returns of the underlying asset is not directly observable, but must be estimated from historical data, from scenario analysis, or from the prices of other options.

20. A hedge ratio of 0.70 implies that a hedged portfolio should consist of

- A. long 0.70 calls for each short stock.
- B. short 0.70 calls for each long stock.
- C. long 0.70 shares for each short call.
- D. long 0.70 shares for each long call.
- E. None of these is correct.

The hedge ratio is the slope of the option value as a function of the stock value. A slope of 0.70 means that as the stock increases in value by \$1, the option increases by approximately \$0.70. Thus, for every call written, 0.70 shares of stock would be needed to hedge the investor's portfolio.

## PART 2 (OPEN QUESTIONS; 60 points at maximum)

### QUESTION 1. (15 points) Equilibrium pricing models

#### Part a. (3 points)

Give the formula of the CAPM and explain its notation. What are the assumptions underlying the CAPM? How do they relate to empirical evidence?

The assumptions are:

- (a) The market is composed of many small investors, who are price-takers; i. e., perfect competition. In reality this assumption was fairly realistic until recent years when institutional investors increasingly began to influence the market with their large transactions.
- (b) All investors have the same holding period. Obviously, different investors have different goals, and thus have different holding periods.
- (c) Investments are limited to those that are publicly traded. In addition, it is assumed that investors may borrow or lend any amount at a fixed, risk-free rate. Obviously, investors may purchase assets that are not publicly traded; however, the dollar volume of publicly traded assets is considerable. The assumption that investors can borrow or lend any amount at a fixed, risk-free rate obviously is false. However, the model can be modified to incorporate different borrowing and lending rates.
- (d) Investors pay no taxes on returns and incur no transaction costs. Obviously, investors do pay taxes and do incur transaction costs.
- (e) All investors are mean-variance efficient. This assumption implies that all investors make decisions based on maximizing returns available at an acceptable risk level; most investors probably make decisions in this manner. However, some investors are pure wealth maximizers (regardless of the risk level); and other investors are so risk averse that avoiding risk is their only goal.
- (f) All investors have homogeneous expectations, meaning that given the same data all investors would process the data in the same manner, resulting in the same risk/return assessments for all investment alternatives.

#### Part b. (2 points)

Consider the multifactor APT with two factors. The risk premiums on the factor 1 and factor 2 portfolios are 5% and 6%, respectively. Stock A has a beta of 1.2 on factor 1, and a beta of 0.7 on factor 2. The expected return on stock A is 17%.

If no arbitrage opportunities exist, what should the risk-free rate of return be?

$$17\% = x\% + 1.2(5\%) + 0.7(6\%); x = 6.8\%.$$



**Part c. (3 points)**

Security A has a beta of 1.0 and an expected return of 12%.

Security B has a beta of 0.75 and an expected return of 11%.

The risk-free rate is 6%.

Explain the arbitrage opportunity that exists; explain how an investor can take advantage of it. Give specific details about how to form the portfolio, what to buy and what to sell (*Hint: form a portfolio of security A and the risk-free asset*).

An arbitrage opportunity exists because it is possible to form a portfolio of security A and the risk-free asset that has a beta of 0.75 and a different expected return than security B. The investor can accomplish this by choosing .75 as the weight in A and .25 in the risk-free asset. This portfolio would have  $E(r_p) = 0.75(12\%) + 0.25(6\%) = 10.5\%$ , which is less than B's 11% expected return. The investor should buy B and finance the purchase by short selling A and borrowing at the risk-free asset.

**Part d. (3 points)**

Consider the regression equation:

$$r_{it} - r_{ft} = g_0 + g_1 b_i + g_2 s^2(e_i) + e_{it}$$

where:  $r_{it} - r_{ft}$  = the average difference between the monthly return on stock  $i$  and the monthly risk-free rate,  $b_i$  = the beta of stock  $i$ ,  $s^2(e_i)$  = a measure of the nonsystematic variance of the stock  $i$ .

If you estimated this regression equation and the CAPM was valid, what would you expect the estimated coefficients  $g_0$ ,  $g_1$ , and  $g_3$  to be? Why?

$g_0 = 0$  (In this model, the coefficient,  $g_0$  represents the excess return of the security, which would be zero if the CAPM held)

$g_1$  equal to the average difference between the monthly return on the market portfolio and the monthly risk-free rate (i.e. the market risk premium)

$g_2 = 0$  (If the CAPM is valid, the excess return on the stock is predicted by the systematic risk of the stock and the excess return on the market, not by the nonsystematic risk of the stock)

**Part e. (4 points)**

Consider the Fama-French 3-factor model:

$$r_i = \alpha_i + \beta_i r_M + \gamma_i SMB + \delta_i HML + e_i$$

where  $r_i$  is the return of a stock  $i$ ,  $r_M$  is the market return, SMB is a factor that proxies for size, and HML – for value. The variances of the three factors are respectively  $\sigma_M^2$ ,  $\sigma_{SMB}^2$  and  $\sigma_{HML}^2$ . The variance of the idiosyncratic source of risk is  $\sigma^2(e)$  and it is the same for all stocks  $i$ .

Assume that the idiosyncratic sources of risk are uncorrelated, and that the three factors are uncorrelated as well.

- i. Give an expression for the systematic risk of stock i in terms of the variances of the three factors.

$$\sigma_i^2 = \beta_i^2 \sigma_M^2 + \gamma_i^2 \sigma_{SMB}^2 + \delta_i^2 \sigma_{HML}^2$$

- ii. Construct an equally weighted portfolio of 10 stocks. What is its non-systematic risk component? Compare it to the non-systematic risk component of the individual stocks.

$$\sigma_P^2 = \frac{1}{10} \sigma_e^2$$

- iii. Consider three stocks,  $i=\{1,2,3\}$ . Construct a portfolio out of the three stocks that has zero exposure to the size factor, and an exposure of 1 to the value factor. Provide the system of equations to be used to solve for the weights. You do not need to find the explicit solution for the weights.

The weights solve the following system of equations:

$$\begin{aligned} w_1 + w_2 + w_3 &= 1 \\ \gamma_1 w_1 + \gamma_2 w_2 + \gamma_3 w_3 &= 0 \\ \delta_1 w_1 + \delta_2 w_2 + \delta_3 w_3 &= 1 \end{aligned}$$

## QUESTION 2. (15 points) Portfolio construction and performance measurement

### Part a. (6 points)

Consider two risky assets, A and B. Denote by  $r_A$  the return of risky asset A, and  $r_B$  the return of risky asset B. Their respective standard deviations are  $\sigma_A$  and  $\sigma_B$ . The two assets are correlated and their correlation coefficient is given by  $\rho$ .

Further, consider a portfolio of the two assets, with weights  $\omega$  and  $(1-\omega)$  respectively.

- i. Give an analytic expression of the return of the portfolio ( $r_p$ ) and its variance ( $\sigma_p^2$ ). What is the expected return of the portfolio  $E[r_p]$ ?

$$\begin{aligned} r_p &= \omega r_A + (1 - \omega) r_B \\ \sigma_p^2 &= \omega^2 \sigma_A^2 + (1 - \omega)^2 \sigma_B^2 + 2\omega(1 - \omega) \sigma_A \sigma_B \rho \end{aligned}$$

- ii. If the correlation between the two assets is  $\rho = -1$ , solve for the weights of a perfectly hedged portfolio.

A perfectly hedged portfolio has a variance of zero. Thus:

$$\begin{aligned} \sigma_p^2 &= \omega^2 \sigma_A^2 + (1 - \omega)^2 \sigma_B^2 - 2\omega(1 - \omega) \sigma_A \sigma_B = (\omega \sigma_A - (1 - \omega) \sigma_B)^2 = 0 \\ \omega \sigma_A - (1 - \omega) \sigma_B &= 0 \\ \omega &= \sigma_B / (\sigma_A + \sigma_B) \end{aligned}$$

- iii. Consider an investor who has the following utility function:

$$U = E[r_p] - \frac{1}{2}A\sigma_p^2$$

where A is the level of risk aversion of the investor. As well, the risk-free return is given by  $r_f$ .

Which value of A makes the investor indifferent between investing in the risky portfolio P and the risk-free asset? Give an analytic expression.

$$\begin{aligned} U(r_p) &= U(r_f) \\ E[r_p] - \frac{1}{2}A\sigma_p^2 &= r_f \\ A &= \frac{2(E[r_p] - r_f)}{\sigma_p^2} \end{aligned}$$

### Part b. (6 points)

The following data are available relating to the performance of Monarch Stock Fund and the market portfolio:

	Monarch	Market Portfolio
Average Return	16%	12%
Standard Deviation of Returns	26%	22%
Beta	1.15	1.00
Residual Standard Deviation	1%	0%

The risk-free return during the sample period was 4%.

- i. Calculate Sharpe's measure of performance for Monarch Stock Fund

$$(16 - 4)/26 = .46$$

- ii. Calculate Jensen's measure of performance for Monarch Stock Fund.

$$\alpha_p = 16\% - [4\% + 1.15(12\% - 4\%)] = 2.8\%;$$

- iii. What is the information ratio measure of performance evaluation for Monarch Stock Fund?

$$\alpha_p/\sigma(e_p) = 2.8\%/1\% = 2.8, \text{ or } 280\%.$$

- iv. If you wanted to evaluate the Seminole Fund using the  $M^2$  measure, what percent of the adjusted portfolio would need to be invested in T-Bills? Calculate the  $M^2$  measure.

$$\begin{aligned} 22/26 &= 0.846 \text{ or } 84.6\% \text{ invested in the Monarch Stock fund, and } 1-84.6\% = 15.4\% \text{ in T-Bills.} \\ M^2 &= [0.846 * 16 + 15.4 * 4] - 12 \end{aligned}$$

**Part c. (3 points)**

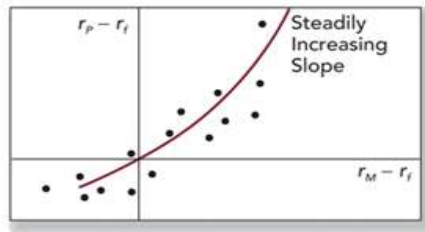
You are evaluating the market timing ability of a portfolio manager. In order to do so, you use the following regression equation:

$$r_p - r_f = a + b(r_M - r_f) + c(r_M - r_f)^2 + e_p$$

where  $r_p$  is the return of the portfolio,  $r_f$  is the risk-free rate,  $r_M$  is the market return, and  $e_p$  is the error term.

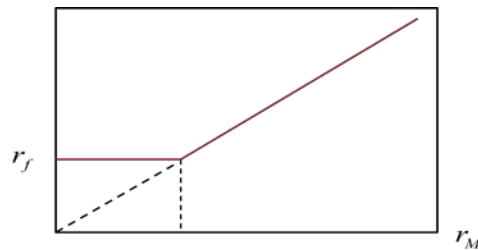
- i. How would you test for the timing ability of the portfolio manager, using the above regression equation? Give also a graphic interpretation.

Test for  $c > 0$  which would indicate market timing ability.



- ii. How would you define an ideal situation of perfect foresight (also give a graphic interpretation)? Using this, how could you compute the value of market timing?

Perfect foresight is equivalent to holding a call option on the index portfolio.



**QUESTION 3. (15 points) Fixed Income****Part a. (4 points)**

You are given the following three coupon bonds with annual coupon payments and face value of 100.

Bond	Maturity	Coupon	Price	Yield to maturity	Notional
A	2	2%	\$ 100.9897	1.4940 %	\$ 100
B	2	6%	\$ 108.8366	1.4831 %	\$ 100
C	3	4%	\$ 106.7183	1.6847 %	\$ 100

Compute the zero rates  $z_1, z_2, z_3$  (up to two decimal places) for maturities of 1, 2, and 3 years.

The prices of bond A and B can be expressed as:

$$P_A = \frac{2}{1+z_1} + \frac{102}{(1+z_2)^2} \text{ and } P_B = \frac{6}{1+z_1} + \frac{106}{(1+z_2)^2}$$

Note that a portfolio of a long position in 3 type A bonds and a short position in one type B bond is a zero-coupon bond, so that:  $3P_A - P_B = \frac{200}{(1+z_2)^2}$

$$\text{From there we solve for } z_2: (1+z_2)^2 = \frac{200}{3 \times 100.9897 - 108.8366} \rightarrow z_2 = 1.5\%$$

$$\text{And for } z_1: z_1 = \frac{2}{100.9897 - 102/(1+0.015)^2} = 0.9\%$$

$$\text{With } z_1 \text{ and } z_2 \text{ known, } z_3 \text{ follows from: } \frac{4}{1+0.009} + \frac{4}{(1+0.015)^2} + \frac{104}{(1+z_3)^3} = 106.7183 \rightarrow z_3 = 1.7\%$$

**Part b. (6 points)**

Using the data on the three coupon bonds in Part (a):

- i. Compute the modified duration of the three bonds.

$$\text{Bond A: Duration(A)} = (2/(1+0.01494) * 1 + 102/(1+0.01494)^2 * 2)/100.9897 = 1.98$$

$$\text{Modified duration(A)} = 1.98/(1+0.01494) = 1.95$$

Bond B:  $\text{Duration}(B) = (6/(1+0.014831) * 1 + 106/(1+0.014831)^2 * 2)/108.8366 = 1.95$

Modified duration(B) =  $1.95/(1+0.014831) = 1.92$

Bond C:  $\text{Duration}(C) = (4/(1+0.016847) * 1 + 4/(1+0.016847)^2 * 2 + 104/(1+0.016847)^3 * 3)/106.7183 = 2.89$

Modified duration(C) =  $2.89/(1+0.016847) = 2.84$

- ii. Construct a portfolio that consists of a long position in 10 type A bonds, a short position in 30 type B bonds and a long position in 50 type C bonds. Obtain the value of the portfolio, and the weights of each bond.

The value of the portfolio:  $10*100.9897 - 30*108.8366 + 50*106.7183 = 3080.713$

Weight of bond A:  $10*100.9897/3080.713 = 0.3278$

Weight of bond B:  $-30*108.8366/3080.713 = -1.0599$

Weight of bond C:  $50*106.7183/3080.713 = 1.732$

- iii. Compute the modified duration of this portfolio.

$0.3278*1.95 - 1.0599*1.92 + 1.732*2.84 = 3.53$

- iv. If you expect an upward shift in the term structure (i.e. an increase of all interest rates) by 50 basis points, what would be the percentage change in the value of the portfolio, based on duration approximation? If alternatively you expect a decrease by 200 basis points, what would be the percentage change in the value of the portfolio? Comment on the quality of the approximation in both cases.

Percentage change in the value of the portfolio for 50bps:  $-3.53*0.5\% = -1.77\%$

Percentage change in the value of the portfolio for -200bps:  $-3.53*(-2\%) = 7.06\%$

The approximation error should be smaller for the 50bps shift.

### Part c. (2 points)

The zero rates for maturities of 1, 2, and 3 years are given by  $z_1$ ,  $z_2$ , and  $z_3$ . Obtain (analytically) the corresponding one-year forward rates for  $t=1, 2, 3$  ( $f_1$ ,  $f_2$ ,  $f_3$ ) in terms of the zero rates.

Express the price of a 3-year coupon bond with annual coupon payments of  $C$  and face value of 100 using the forward rates.

The forward rates from the zero curve:

$$f_1 = z_1$$

$$f_2 = \frac{(1 + z_2)^2}{1 + z_1} - 1$$

$$f_3 = \frac{(1 + z_3)^3}{(1 + z_2)^2} - 1$$

The price of a coupon bond and 3 years maturity using the forward rates:

$$P = \frac{C}{1 + f_1} + \frac{C}{(1 + f_1)(1 + f_2)} + \frac{C + 100}{(1 + f_1)(1 + f_2)(1 + f_3)}$$

#### Part d. (3 points)

Assume that you have a portfolio with duration of 1.8 and convexity of 6. The current yield curve is flat at 5%. Construct a portfolio of zero coupon bonds with maturities of 2 and 5 years, and cash, such that it matches the duration and convexity of your portfolio.

The duration of a zero coupon bond is equal to its time to maturity, the durations of the two zero bonds with maturities of 2 and 5 years are 2 and 5 respectively.

The convexity of a zero coupon bond with maturity T, yield y, face value F and price P is:

$$C = \frac{1}{P(1+y)^2} \frac{F}{(1+y)^T} (T^2 + T) = \frac{T(T+1)}{(1+y)^2}$$

(as the price of the zero coupon bond is  $P = F / (1+y)^T$ ).

Then the convexities of the two bonds are 5.44 for the 2-year bond and 27.21 for the 5-year bond.

Then if we hold a in the 2-year zero, b in the 5-year zero and c in cash, we have to solve:

$$a + b + c = 1$$

$$2a + 5b = 1.8$$

$$5.44a + 27.21b = 6$$

We obtain a=0.70, b=0.08, c=0.22.

#### QUESTION 4. (15 points) Option pricing

##### Part a. (3 points)

Discuss the differences in writing covered and naked calls (a naked call refers to writing a call on a stock that the investor does not own). Are risks involved in the two strategies similar or different? Explain.

Writing a covered call is selling a call on stock the investor owns. Thus, this strategy is very conservative; the investor receives the premium income from writing the call. If the call is exercised, the stock is called away from the investor; thus the investor has limited his or her upside potential. Writing a naked call is a very risky strategy. The investor sells a call on a stock the investor does not own. If the price of the stock increases, the option will be exercised and the investor must go into the open market and buy the stock at the prevailing market price. Theoretically, the price to which the stock can increase is unlimited; thus, the investor's potential loss is unlimited.

##### Part b. (2 points)

An American-style call option with six months to maturity has a strike price of \$35. The underlying stock now sells for \$43. The call premium is \$12.

- i. Calculate the intrinsic value of the call  
 $43 - 35 = \$8.$
- ii. What is the time value of the call?  
 $12 - (43 - 35) = \$4.$

##### Part c. (8 points)

Consider the following binomial tree for the evolution of the €/£ exchange rate over the period of 1 year, assuming 2 steps ( $t=0$ ,  $t=1$ ,  $t=2$ ). The current exchange rate at  $t=0$  is  $S_0 = 0.7$  €/£. The annual interest rate is 3%, and the exchange rate can increase by 10% or decrease by 5% each period.

- i. Draw the binomial price tree for ( $t=0$ ,  $t=1$ ,  $t=2$ ).
- ii. Calculate the risk-neutral probability of an upward movement and that of a downward movement.
- iii. Consider a Dutch company that will have to pay \$ 1 million in 1 year. The spot exchange rate today is  $S_0 = 0.70$  €/£. The company wants to hedge itself against an unfavourable foreign exchange rate in 1 year, but would also like to gain from a depreciation of the US dollar. It has the following two options:

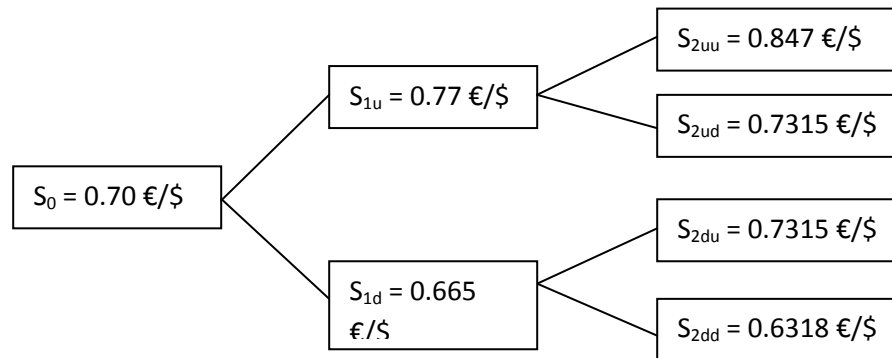


- A. Buy a standard call option with a strike price of  $X = 0.70 \text{ €/}\$$  and expiry in one year;
- B. Buy a down-and-out barrier call option. This option behaves like a standard call option, except that it is terminated and becomes worthless once the foreign exchange rate at any moment during the life of the option becomes equal or lower than the chosen barrier. It has a strike price of  $X = 0.70 \text{ €/}\$$  and it is terminated if the exchange rate becomes equal to or falls below a barrier of  $0.68 \text{ €/}\$$ .

Compute the price of the call option in (A) and of the barrier down-and-out call option in (B) using binomial pricing with 2 steps ( $t=0, t=1, t=2$ ) and the binomial tree from (i).

Is there a difference in the prices of the standard call option and the down-and-out barrier call? Which one would you recommend to the company?

i. The binomial tree:



ii. The tree has two steps, so the periodic interest rate is  $0.03/2 = 0.015$ . Then the risk neutral probability  $Q$  at node  $t=0$ :  $\frac{0.7 \times (1+0.015) - 0.665}{0.77 - 0.665} = 0.43$ . It is the same for the node  $t=1$  (up) and  $t=1$  (down).

iii. (A) The price of the plain vanilla call option can be obtained using the risk-neutral probabilities:

At time  $t=2$  the prices are:

$$\begin{aligned}
 C_{2uu} &= \max(0, 0.847 - 0.7) = 0.147 \\
 C_{2ud} &= C_{2du} = \max(0, 0.7315 - 0.7) = 0.0315 \\
 C_{2dd} &= \max(0, 0.6318 - 0.7) = 0
 \end{aligned}$$

At time  $t=1$  the prices are:

$$\begin{aligned}
 C_{1u} &= \frac{(0.43 \times 0.147 + (1 - 0.43) \times 0.0315)}{0 + 0.015} = 0.0803 \\
 C_{1d} &= \frac{(0.43 \times 0.0315 + (1 - 0.43) \times 0)}{0 + 0.015} = 0.0134
 \end{aligned}$$

At time  $t=0$  the call price is then:

$$C_0 = \frac{(0.43 \times 0.0803 + (1 - 0.43) \times 0.0134)}{0 + 0.015} = 0.0418$$

(B) The price of the barrier option depends on whether the exchange rate has fallen below 0.68 €//\$ during its life. At time  $t=1$  in the down node (1d) the exchange rate is 0.665 €//\$ and thus the barrier option is terminated at this node (i.e. before maturity). It will have a value of zero at that node, as well as at the following nodes 2du and 2dd. Its price at nodes 2uu and 2ud is identical to the one in (i) for these nodes. Thus:

$$C_{2uu} = 0.147, C_{2ud} = 0.0315$$

$$C_{2du} = C_{2dd} = 0 \text{ (as exchange rate below barrier at } t=1)$$

At time  $t=1$ :  $C_{1u} = 0.0803, C_{1d} = 0$  (as exchange rate below barrier)

At time  $t=0$ , the price of the barrier call is then:

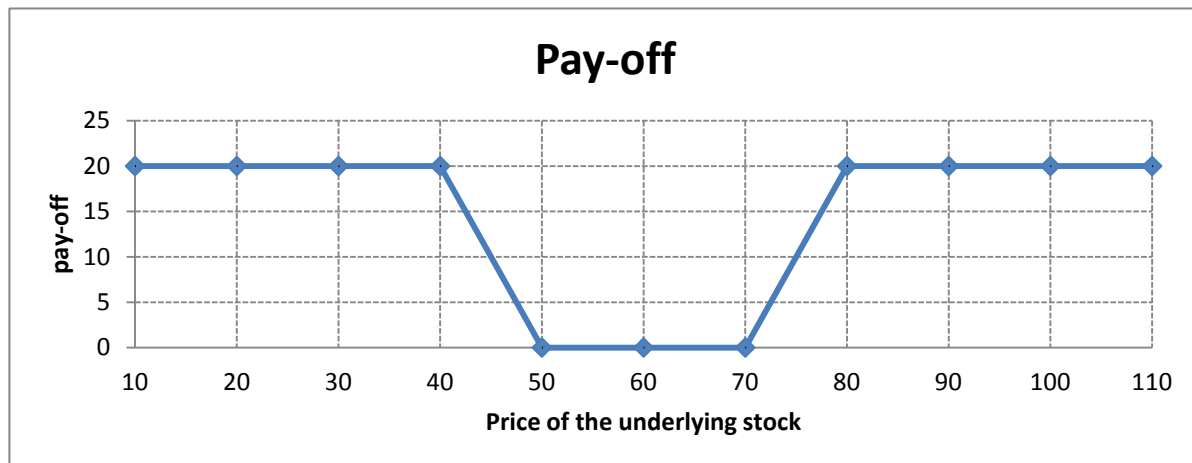
$$C_0 = \frac{(0.43 \times 0.0803 + (1 - 0.43) \times 0)}{0 + 0.015} = 0.0343$$

The barrier call has a lower premium than the standard call option. It still protects the buyer of the option against unfavorable movements of the spot exchange rate but it is terminated once the spot rate reaches the barrier of 0.68 €//\$. However, at that moment the company can close its exposure earlier at this favorable spot rate.

#### Part d. (2 points)

Replicate the pay-off structure below (known as 'short iron condor') using call or put options. On the horizontal axis you have the price of the underlying stock, and on the vertical axis – the pay-off at maturity.

NB. Mind the scale on the horizontal and the vertical axis.



Long 2 calls @ 70, long 2 puts @ 50, short 2 calls @ 80, short 2 puts @ 40