

Exam: Investments 3.4

Code: 60332090

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Duration: 2 hours and 45 minutes

Calculator allowed: Yes

Graphical calculator allowed: Yes

Number of questions: 20 multiple choice questions and 4 open questions

Type of questions: Open/ multiple choice

Answer in: English

Remarks: Be concise and complete in your answers (including calculations). Always explain your answers, even if not explicitly called for. Use your time efficiently, using the maximum number of points per question as a guideline.

Credit score: The maximum possible scores for each part and question are indicated. In total, you can earn 100 points. Your final exam grade is determined by dividing the number of points by 10.

Grades: The grades will be made public on: May 30 2011.

Inspection: Tuesday, May 31 2011 at 10.00 in room 1E-67.

Number of pages: 12 (including front page)

Good luck!

PART 1 (MULTIPLE CHOICE; 40 points at maximum)

Read the questions and answers carefully and write down your answer on your answer sheet. Your final score is determined as (# correct answers - 2) * 40/18. Negative scores for this part of the exam are set to zero.

1. The risk premium for common stocks

- A. cannot be zero, for investors would be unwilling to invest in common stocks.
- B. must always be positive, in theory.
- C. is negative, as common stocks are risky.
- D.** A and B.
- E. A and C.

If the risk premium for common stocks were zero or negative, investors would be unwilling to accept the lower returns for the increased risk.

2. An investor invests 40 percent of his wealth in a risky asset with an expected rate of return of 0.17 and a variance of 0.08 and 60 percent in a T-bill that pays 4.5 percent. His portfolio's expected return and standard deviation are _____ and _____, respectively.

- A. 0.114; 0.126
- B. 0.087; 0.068
- C.** 0.095; 0.113
- D. 0.087; 0.124
- E. none of the above

$$E(r_p) = 0.4(17\%) + 0.6(4.5\%) = 9.5\%; s_p = 0.4(0.08)^{1/2} = 11.31\%.$$

3. The efficient frontier of risky assets is

- A.** the portion of the investment opportunity set that lies above the global minimum variance portfolio.
- B. the portion of the investment opportunity set that represents the highest standard deviations.
- C. the portion of the investment opportunity set which includes the portfolios with the lowest standard deviation.
- D. the set of portfolios that have zero standard deviation.
- E. both A and B are true.

Portfolios on the efficient frontier are those providing the greatest expected return for a given amount of risk. Only those portfolios above the global minimum variance portfolio meet this criterion.

4. Consider the single-index model. The alpha of a stock is 0%. The return on the market index is 16%. The risk-free rate of return is 5%. The stock earns a return that exceeds the risk-free rate by 11% and there are no firm-specific events affecting the stock performance. The β of the stock is _____.

- A. 0.67
- B. 0.75
- C. 1.0**
- D. 1.33
- E. 1.50

$$11\% = 0\% + b(11\%); b = 1.0.$$

5. According to the Capital Asset Pricing Model (CAPM), under priced securities

- A. have positive betas.
- B. have zero alphas.
- C. have negative betas.
- D. have positive alphas.**
- E. none of the above.

According to the Capital Asset Pricing Model (CAPM), under priced securities have positive alphas.

6. The capital asset pricing model assumes

- A. all investors are price takers.
- B. all investors have the same holding period.
- C. investors pay taxes on capital gains.
- D. both A and B are true.**
- E. A, B and C are all true.

The CAPM assumes that investors are price-takers with the same single holding period and that there are no taxes or transaction costs.

7. Consider the multifactor model APT with two factors. Portfolio A has a beta of 0.75 on factor 1 and a beta of 1.25 on factor 2. The risk premiums on the factor 1 and factor 2 portfolios are 1% and 7%, respectively. The risk-free rate of return is 7%. The expected return on portfolio A is _____ if no arbitrage opportunities exist.

- A. 13.5%
- B. 15.0%
- C. 16.5%**
- D. 23.0%
- E. none of the above

$$7\% + 0.75(1\%) + 1.25(7\%) = 16.5\%.$$

8. If you believe in the _____ form of the EMH, you believe that stock prices reflect all information that can be derived by examining market trading data such as the history of past stock prices, trading volume or short interest.

- A. semistrong
- B. strong
- C. weak**
- D. all of the above
- E. none of the above

The information described above is market data, which is the data set for the weak form of market efficiency. The semistrong form includes the above plus all other public information. The strong form includes all public and private information.

9. Consider the regression equation:

$$r_i - r_f = g_0 + g_1 b_i + g_2 s^2(e_i) + e_{it}$$

where:

$r_i - r_f$ = the average difference between the monthly return on stock i and the monthly risk-free rate

b_i = the beta of stock i

$s^2(e_i)$ = a measure of the nonsystematic variance of the stock i

If you estimated this regression equation and the CAPM was valid, you would expect the estimated coefficient, g_2 to be

A. 0

B. 1

C. equal to the risk-free rate of return

D. equal to the average difference between the monthly return on the market portfolio and the monthly risk-free rate

E. none of the above

If the CAPM is valid, the excess return on the stock is predicted by the systematic risk of the stock and the excess return on the market, not by the nonsystematic risk of the stock.

10. A preferred stock will pay a dividend of \$1.25 in the upcoming year, and every year thereafter, i.e., dividends are not expected to grow. You require a return of 12% on this stock. Use the constant growth Dividend Discount Model to calculate the intrinsic value of this preferred stock.

A. \$11.56

B. \$9.65

C. \$11.82

D. \$10.42

E. none of the above

$$1.25 / .12 = 10.42$$

11. Consider two bonds, A and B. Both bonds presently are selling at their par value of \$1,000. Each pays interest of \$120 annually. Bond A will mature in 5 years while bond B will mature in 6 years. If the yields to maturity on the two bonds change from 12% to 10%, _____.

A. both bonds will increase in value, but bond A will increase more than bond B

B. both bonds will increase in value, but bond B will increase more than bond A

C. both bonds will decrease in value, but bond A will decrease more than bond B

D. both bonds will decrease in value, but bond B will decrease more than bond A

E. none of the above

The longer the maturity, the greater the price change when interest rates change.

12 You have just purchased a 12-year zero-coupon bond with a yield to maturity of 9% and a par value of \$1,000. What would your rate of return at the end of the year be if you sell the bond? Assume the yield to maturity on the bond is 10% at the time you sell.

- A. 10.00%
- B. 20.42%
- C. -1.4%**
- D. 1.4%
- E. none of the above

$\$1,000/(1.09)^{12} = \355.53 ; $\$1,000/(1.10)^{11} = \350.49 ; $(\$350.49 - \$355.53)/\$355.53 = -1.4\%$.

13. The **pure yield curve** can be estimated

- A. by using zero-coupon bonds.
- B. by using coupon bonds if each coupon is treated as a separate "zero."
- C. by using corporate bonds with different risk ratings.
- D. by estimating liquidity premiums for different maturities.
- E. A and B.**

The pure yield curve is calculated using zero coupon bonds, but coupon bonds may be used if each coupon is treated as a separate "zero."

14 Holding other factors constant, the duration of a bond is negatively correlated with the bond's

- A. time to maturity.
- B. coupon rate.
- C. yield to maturity.
- D. B and C.**
- E. none of the above.

Duration is negatively correlated with coupon rate and yield to maturity.

15. Immunization through duration matching of assets and liabilities may be ineffective or inappropriate because

- A. conventional duration strategies assume a flat yield curve.
- B. duration matching can only immunize portfolios from parallel shifts in the yield curve.
- C. immunization only protects the nominal value of terminal liabilities and does not allow for inflation adjustment.
- D. both A and C are true.
- E. all of the above are true.**

All of the above are correct statements about the limitations of immunization through duration matching.

16. According to the put-call parity theorem, the value of a European put option on a non-dividend paying stock is equal to:

- A. the call value plus the present value of the exercise price plus the stock price.
- B.** the call value plus the present value of the exercise price minus the stock price.
- C. the present value of the stock price minus the exercise price minus the call price.
- D. the present value of the stock price plus the exercise price minus the call price.
- E. none of the above.

$P = C - SO + PV(X)$, where SO = the market price of the stock, and X = the exercise price.

17. Which of the following factors affect the price of a stock option

- A. the risk-free rate.
- B. the riskiness of the stock.
- C. the time to expiration.
- D. the expected rate of return on the stock.
- E.** A, B, and C.

A, B, and C are directly related to the price of the option; D does not affect the price of the option.

18. All the inputs in the Black-Scholes Option Pricing Model are directly observable **except**

- A. the price of the underlying security.
- B. the risk free rate of interest.
- C. the time to expiration.
- D.** the variance of returns of the underlying asset return.
- E. none of the above.

The variance of the returns of the underlying asset is not directly observable, but must be estimated from historical data, from scenario analysis, or from the prices of other options.

19. A hedge ratio of 0.70 implies that a hedged portfolio should consist of

- A. long 0.70 calls for each short stock.
- B. short 0.70 calls for each long stock.
- C. long 0.70 shares for each short call.**
- D. long 0.70 shares for each long call.
- E. none of the above.

The hedge ratio is the slope of the option value as a function of the stock value. A slope of 0.70 means that as the stock increases in value by \$1, the option increases by approximately \$0.70. Thus, for every call written, 0.70 shares of stock would be needed to hedge the investor's portfolio.

20. If interest rate parity holds

- A. covered interest arbitrage opportunities will exist
- B. covered interest arbitrage opportunities will not exist**
- C. arbitrageurs will be able to make risk-free profits
- D. A and C
- E. B and C

If interest rate parity holds covered interest arbitrage opportunities will not exist

PART 2 (OPEN QUESTIONS; 60 points at maximum)

QUESTION 1. (15 points) Equilibrium pricing models

Part a. (3 points)

Explain the separation property of a portfolio selection problem.

The portfolio choice problem may be separated into two independent tasks: determination of the optimal portfolio that does not depend on individual investor's preferences and allocation of the portfolio between risk-free and risky assets, which depends on the risk aversion of the investor.

Part b. (3 points)

Give the equation of the single factor model. Explain the decomposition of the risky asset's return and of its variance.

The risky asset's return can be decomposed into an expected and an unexpected part:

$r_i = E(r_i) + \beta_i \cdot m + e_i$, where m is a common factor and e_i is the unexpected return.

Similarly, the variance of the risky asset has two components – systematic and firm specific:

$\sigma_i^2 = \beta_i^2 \cdot \sigma_m^2 + \sigma^2(e_i)$, where σ_m^2 is the variance of the common factor and $\sigma^2(e_i)$ is the variance of the firm-specific factor.

Part c. (4 points)

Assume that the CAPM holds. The risk free rate r_f is 5% and the market return r_M is 8%. Consider the following two stocks:

Stocks	β	P/E
A	1.5	14.29
B	0.8	13.51

β - market beta of each stock

P/E – price-earnings ratio

The retention ratio (**b**) for the two stocks is 0.5.

- Compute the return on equity (ROE) of the two stocks, assuming a constant growth model with endogenous earnings growth (round at the third decimal).

$$k_A = 0.05 + 1.5 \cdot (0.08 - 0.05) = 0.095, \quad k_B = 0.05 + 0.8 \cdot (0.08 - 0.05) = 0.074$$

$$ROE_A = (14.29 \cdot 0.095 - 0.5) / (14.29 \cdot 0.5) = 0.12$$

$$ROE_B = (13.51 \cdot 0.074 - 0.5) / (13.51 \cdot 0.5) = 0.074$$

- Estimate the growth rate for both firms.

$$g_A = 0.12 \cdot 0.5 = 0.06, g_B = 0.074 \cdot 0.5 = 0.037$$

- iii. What would the effect on the P/E ratio be if you change the retention ratio of stock B, keeping ROE constant? Why?

It will not change since $ROE_B = k_B$.

Part d. (5 points)

Consider the CAPM model augmented with a liquidity factor:

$$r_i = \alpha_i + \beta_i r_M + \gamma_i L + e_i$$

where r_i is the return of a stock i , r_M is the market return, L is a liquidity factor. The variances of the two factors are respectively σ_M^2 and σ_L^2 . The variance of the idiosyncratic source of risk is $\sigma^2(e)$ for all stocks i .

Assume that the idiosyncratic sources of risk are uncorrelated, and that the factors are uncorrelated as well.

Consider two stocks ($i=\{1, 2\}$).

- i. Give an expression for the systematic risk of each of the two stocks.

$$\sigma_i^2 = \beta_i^2 \sigma_M^2 + \gamma_i^2 \sigma_L^2$$

- ii. Construct an equally weighted portfolio of the two stocks. What is its non-systematic risk component? Compare it to the non-systematic risk component of the individual stocks.

$$\sigma_P^2 = \frac{1}{2} \sigma_e^2$$

- iii. Construct a portfolio out of the two stocks that has zero exposure to the liquidity factor. Give an analytic expression of its weights.

The weights solve the following system of equations:

$$w_1 + w_2 = 1$$

$$\gamma_1 w_1 + \gamma_2 w_2 = 0$$

With the solution: $w_1 = \frac{\gamma_2}{\gamma_2 - \gamma_1}$ and $w_{12} = -\frac{\gamma_1}{\gamma_2 - \gamma_1}$.

QUESTION 2. (15 points) Portfolio construction and performance measurement

Part a. (4 points)

Consider an investor with the following utility function: $U = E(r) - (A/2)s^2$, where $E(r)$ is the expected return of a portfolio and s^2 is its variance.

- i. Explain the significance of A in the above utility function

A is a scale factor that indicates the investor's degree of risk aversion. The higher the value of A , the more risk averse the investor.

- ii. A portfolio has an expected rate of return of 0.15 and a standard deviation of 0.15. The risk-free rate is 6 percent. Which value of A makes this investor indifferent between the risky portfolio and the risk-free asset?

$U(R_f) = 6$. Then, in order for the investor to be indifferent between the risky portfolio and the risky asset, the following must hold: $0.06 = 0.15 - A/2(0.15)^2$; $0.06 - 0.15 = -A/2(0.0225)$; $-0.09 = -0.01125A$; $A = 8$.

- iii. Consider the following 4 opportunities for investment:

Investment	Expected Return $E(r)$	Standard Deviation
1	0.12	0.3
2	0.15	0.5
3	0.21	0.16
4	0.24	0.21

Based on the utility function above and for $A=4$, which investment would you select? Which investment would you select if you were risk neutral? Why?

If you are risk averse and $A=4$: $U(3) = 0.21 - 4/2(0.16)^2 = 15.88$ (highest utility of choices). If you are risk neutral, your only concern is with return, not risk, thus you would select the opportunity with the highest return, i.e. 4.

Part b. (4 points)

Consider the following probability distribution for stocks C and D:

State	Probability	Return on Stock C	Return on Stock D
1	0.30	7%	-9%
2	0.50	11%	14%
3	0.20	-16%	26%

- i. Calculate the expected rates of return of stocks C and D.

$$E(R_C) = 0.30(7\%) + 0.5(11\%) + 0.20(-16\%) = 4.4\%;$$

$$E(R_D) = 0.30(-9\%) + 0.5(14\%) + 0.20(26\%) = 9.5\%.$$

- ii. Calculate the standard deviations of the two stocks.

$$s_C = [0.30(7\% - 4.4\%)^2 + 0.5(11\% - 4.4\%)^2 + 0.20(-16\% - 4.4\%)^2]^{1/2} = 10.35\%;$$

$$s_D = [0.30(-9\% - 9.5\%)^2 + 0.5(14\% - 9.5\%)^2 + 0.20(26\% - 9.5\%)^2]^{1/2} = 12.93\%.$$

- iii. Calculate the coefficient of correlation between C and D.

$$\text{cov}_{C,D} = 0.30(7\% - 4.4\%)(-9\% - 9.5\%) + 0.50(11\% - 4.4\%)(14\% - 9.5\%) + 0.20(-16\% - 4.4\%)(26\% - 9.5\%) = -0.00669;$$

$$\text{correlation}_{C,D} = -0.00669 / [(0.1035)(0.1293)] = -0.5.$$

- iv. If you invest 25% of your money in C and 75% in D, what would be your portfolio's expected rate of return and standard deviation?

$$E(R_p) = 0.25(4.4\%) + 0.75(9.5\%) = 10.425\%;$$

$$s_p = [(0.25)^2(10.35\%)^2 + (0.75)^2(12.93\%)^2 + 2(0.25)(0.75)(10.35)(12.93)(-0.5)]^{1/2} = 8.7\%.$$

Part c. (4 points)

You want to evaluate three mutual funds. The risk-free return during the sample period is 6%. The average returns, standard deviations, and betas for the three funds are given below, in addition to information regarding the S&P 500 index.

	<u>Average Return</u>	<u>Standard. Deviation</u>	<u>Beta</u>
Fund A	13%	10%	0.5
Fund B	19%	20%	1.0
Fund C	25%	30%	1.5
S&P 500	18%	16%	1.0

- i. Rank the three funds using the Sharpe measure.

$$A: (13\% - 6\%)/10\% = 0.7; B: (19\% - 6\%)/20\% = 0.65; C: (25\% - 6\%)/30\% = 0.63; \text{S\&P 500: } (18\% - 6\%)/16\% = 0.75.$$

- ii. Define the Treynor measure. How does it differ from the Sharpe measure? Rank the funds according to it.

The Treynor measure gives the excess return per unit of systematic risk ($T = (r_p - r_f)/\beta_p$), while the Sharpe ratio is the excess return per unit of risk.

$$A: (13\% - 6\%)/0.5 = 14; B: (19\% - 6\%)/1.0 = 13; C: (25\% - 6\%)/1.5 = 12.7; \text{S\&P 500: } (18\% - 6\%)/1.0 = 12.$$

Part d. (3 points)

You have a time series of realized returns of a mutual fund, a stock and a bond index, and treasuries. You want to perform style analysis of the mutual fund. Give an equation that would allow you to do that and explain its components. Explain how you would estimate its coefficients.

$$R_t = \alpha + \beta_1 \text{Stocks}_t + \beta_2 \text{Bonds}_t + \beta_3 \text{Treasuries}_t + e_t$$

Where R_t is the excess return of the fund, α indicates the abnormal return of the fund over the period, and the style funds are the excess returns of the stock and the bond index, as well as the treasuries. The beta coefficients should sum up to one in order to represent the weights in the style

funds. The factor exposures give the return due to a style. The factor loadings can be estimated by minimizing the sum of squared residuals.

QUESTION 3. (15 points) Fixed Income

Part a. (3 points)

You purchased an annual interest coupon bond one year ago with 6 years remaining to maturity at the time of purchase. The coupon interest rate is 10% and par value is \$1,000. At the time you purchased the bond, the yield to maturity was 8%.

If you sell the bond after receiving the first interest payment and the bond's yield to maturity has changed to 7%, what would be your annual total rate of return on holding the bond for that year?

$FV = 1000$, $PMT = 100$, $n = 6$, $i = 8$, $PV = 1092.46$; $FV = 1000$, $PMT = 100$, $n = 5$, $i = 7$, $PV = 1123.01$; $HPR = (1123.01 - 1092.46 + 100) / 1092.46 = 11.95\%$.

Part b. (3 points)

Suppose that all investors expect that interest rates for the 4 years will be as follows:

Year	Forward Interest Rate
0	(today) 5%
1	7%
2	9%
3	10%

- What is the price of 3-year zero coupon bond with a par value of \$1,000?
 $\$1,000 / (1.05)(1.07)(1.09) = \816.58
- What is the price of a 2-year maturity bond with a 10% coupon rate paid annually? (Par value = \$1,000)
 $y = [(1.05)(1.07)]^{1/2} - 1 = 6\%$; $FV = 1000$, $n = 2$, $PMT = 100$, $PV = \$1,073.34$

Part c. (4 points)

You are given the following three coupon bonds with annual coupon payments and face value of 100.

Bond	Maturity	Coupon	Price	Yield to maturity	Face value

A	2	2%	\$ 98.12	2.98%	\$ 100
B	2	8%	\$ 109.72	2.93%	\$ 100
C	3	7%	\$ 110.04	3.42%	\$ 100

Compute the zero rates z_1, z_2, z_3 (up to two decimal places) for maturities of 1, 2, and 3 years.

The prices of bond A and B can be expressed as: $P_A = \frac{2}{1+z_1} + \frac{102}{(1+z_2)^2}$ and $P_B = \frac{8}{1+z_1} + \frac{108}{(1+z_2)^2}$.

Notice that a portfolio of a long position in 4 A bonds and a short position in one B bond has the same price as that of 3 zero-coupon bonds with maturity of 2 years, so that $4P_A - P_B = \frac{300}{(1+z_2)^2}$, from where $z_2 = 3\%$. Then $z_1 = 0.93\%$.

With z_1 and z_2 known, z_3 follows from: $P_C = \frac{7}{1+z_1} + \frac{7}{(1+z_2)^2} + \frac{107}{(1+z_3)^3}$, $z_3 = 3.5\%$.

Part d. (5 points)

You have a callable bond position. Its modified duration is 15 and its convexity is 110. As well, you have 3 zero coupon bonds with maturities of 5, 10 and 20 years. The face value of the zero coupon bonds is \$100, and the yield curve is flat at 5%.

- i. Calculate the modified duration and the convexity of the three zero coupon bonds.

The durations of the three zero coupon bonds are equal to their maturities, and their modified durations are respectively $D_1 = 5/(1+0.05) = 4.76$, $D_2 = 10/(1+0.05) = 9.52$, $D_3 = 20/(1+0.05) = 19.05$.

The convexity of a zero coupon bond is equal to: $C = 1/(P*(1+y)^2) * 100*(T^2+T)/(1+y)^T = 1/(P*(1+y)^2) * P*(T^2+T) = (T^2+T)/(1+y)^2$, where T is maturity, P is the price of the bond and y is the yield.

The convexities: $C_1 = (5^2+5)/(1+0.05)^2 = 27.21$, $C_2 = (10^2+10)/(1+0.05)^2 = 99.77$, $C_3 = (20^2+20)/(1+0.05)^2 = 380.95$.

- ii. Construct a portfolio of the zero coupon bonds to hedge the callable bond position by matching its modified duration. As the combination of zero coupon bonds is not unique, give one possible portfolio that matches the duration of the callable bond.

The portfolio weights in the three zero coupon bonds are w_1, w_2 , and $(1-w_1-w_2)$. To match the callable bond modified duration, one has the following constraint:

$$4.76*w_1 + 9.52*w_2 + 19.05*(1-w_1-w_2) = 15.$$

As it has more than one solution, one possibility is to fix $w_1=0.2$, then solve for w_2 , which yields $w_2 = 0.125$, and thus the third bond's weight should be 0.675.

- iii. Suggest a way to select the best combination of zero coupon bonds to hedge the callable bond exposure. Do not provide a solution, but state the equation (system of equations) needed.

A best solution could be obtained by also matching the convexity, thus solving the following system of equations:

$$4.76 \cdot w_1 + 9.52 \cdot w_2 + 19.05 \cdot (1 - w_1 - w_2) = 15.$$

$$27.21 \cdot w_1 + 99.77 \cdot w_2 + 380.95 \cdot (1 - w_1 - w_2) = 110.$$

QUESTION 4. (15 points) Option pricing

Part a. (3 points)

You plan to invest \$1000 one year from now at a 1-year forward rate. Your pay-off is presented in the table below:

Year 1	0
Year 2	-1000
Year 3	$1000(1+f_3)$

where f_3 denotes the 1-year FWD rate in year 3.

- i. What is the parity relationship that exists between the forward rate f_3 and the yields of 2-year and 3-year zero coupon bonds, denoted by y_2 and y_3 ?

$$1+f_3 = (1+y_3)^3 / (1+y_2)^2.$$

- ii. Construct a strategy using 2 zero-coupon bonds with maturity in year 2 and year 3 that can replicate this pay-off structure. The prices today (at year 1) of the two bonds are P_2 and P_3 respectively and their face values are \$1000. What are the proportions that you will invest in each one of the zero-coupon bonds?

At year 1 – short one 2-year zero-coupon bond and long P_2/P_3 fraction of a 3-year zero-coupon bond. The pay-off structure:

$$\text{Year 1: } P_2 - P_2/P_3 \cdot P_3 = 0$$

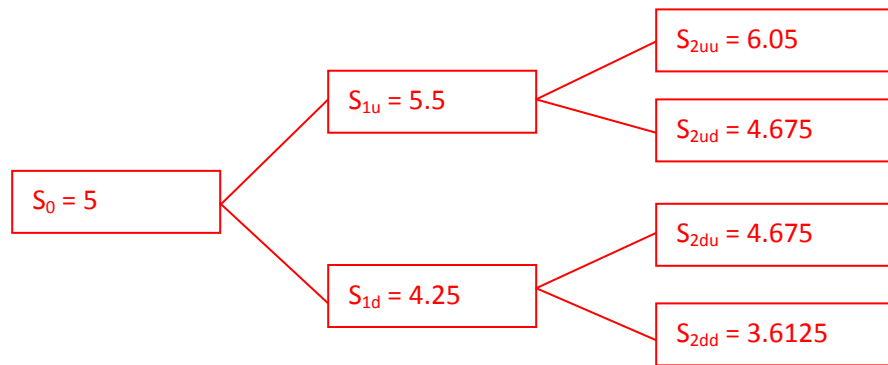
$$\text{Year 2: } -1000$$

$$\text{Year 3: } P_2/P_3 \cdot 1000 = 1000 \cdot (1+f_3)$$

Part b. (2 points)

Consider a 2-period binomial tree ($t=0, t=1, t=2$) that describes the price evolution of stock A over 1 year. The price today is \$5, and it can move up and down each period with a factor of 1.1 and 0.85 respectively.

- i. Draw a diagram of the binomial tree and calculate the stock price at each node.



- ii. Calculate the risk-neutral probability of an upward movement if the annual interest rate is 3%. (Hint: obtain the periodic risk-free rate first).

The periodic risk-free rate $r_f = 0.03/2 = 0.015$.

The risk-neutral probability $Q = (S_0 \cdot (1+r_f) - S_d) / (S_u - S_d) = (5 \cdot (1+0.015) - 4.25) / (5.5 - 4.25) = 0.66$.

Part c. (5 points)

Consider a *down-and-out* barrier option on the stock from Part b. It is a path-dependent option that expires if the underlying price falls below a pre-determined knock-out barrier (regardless of whether the price moves back up afterwards or not). The option behaves like a standard one as long as the price of the underlying remains above the barrier.

Consider the binomial tree from Part b. The strike price is $X = \$4.5$, and the barrier price is $\$4.75$. Compute the price of a down-and-out call (note that the up-down and the down-up nodes on the tree at maturity have different implications in this case). As well, calculate the price of a plain vanilla call. Comment on the reason for the difference between both prices.

The down-and-out call: the price at the down node at $t=1$ is $S_{1d}=4.25$, which falls below the knock-out barrier of 4.75. Thus, from this node on the price of the call is zero: $C_d = C_{dd} = C_{du} = 0$. The price $S_{ud} = 4.675 < 4.75$ too, so $C_{ud} = 0$. $C_{uu} = \max(0, 6.05 - 4.5) = 1.55$, and at $t=1$ $C_u = (0.66 \cdot 1.55 + (1-0.66) \cdot 0) / (1+0.015) = 1.0079$. At $t=0$ $C = (0.66 \cdot 1.0079 + (1-0.66) \cdot 0) / (1+0.015) = 0.6554$.

The plain vanilla call: $C_{uu} = \max(0, 6.05 - 4.5) = 1.55$, $C_{ud} = C_{du} = \max(0, 4.675 - 4.5) = 0.175$, $C_{dd} = \max(0, 3.613 - 4.5) = 0$. $C_u = (0.66 \cdot 1.55 + (1-0.66) \cdot 0.175) / (1+0.015) = 1.0665$, $C_d = (0.66 \cdot 0.175 + (1-0.66) \cdot 0) / (1+0.015) = 0.1138$. $C = (0.66 \cdot 1.0665 + (1-0.66) \cdot 0.1138) / (1+0.015) = 0.7316$.

Part d. (5 points)

Construct an options strategy that allows an investor to benefit from large upward or downward price moves of the underlying asset, while limiting his losses otherwise. Assume that the current price is \$20 and that the investor aims at limiting his loss to \$2 when the price of the underlying moves by 10% in either direction. For higher price volatility the investor wants to assure an upward potential.

What options will you use to construct such a strategy? Specify type and strike price.

Draw a profit diagram of the option strategy, considering the premium paid or received for the options.

Long one put at 18, long one call at 22.

