

Exam: Investments 3.4

Code: 60331090

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Calculator allowed: Yes

Graphical calculator allowed: Yes

Number of questions: 20 multiple choice questions and 4 open questions

Type of questions: Open/ multiple choice

Answer in: English

Remarks: Be concise and complete in your answers (including calculations). Always explain your answers, even if not explicitly called for. Use your time efficiently, using the maximum number of points per question as a guideline.

Credit score: The maximum possible scores for each part and question are indicated. In total, you can earn 100 points. Your final exam grade is determined by dividing the number of points by 10.

Grades: The grades will be made public on: April 8 2010.

Inspection: Thursday, April 8 2010 at 10.00 in room 1E-67.

Number of pages: 12 (including front page)

**Good luck and success!**

**PART 1 (MULTIPLE CHOICE; 40 points at maximum)**

**Read the questions and answers carefully and write down your answer on your answer sheet. Your final score is determined as (# correct answers - 2) \* 40/18. Negative scores for this part of the exam are set to zero.**

1. Which of the following statements is (are) **true** regarding the variance of a portfolio of two risky securities?
  - A. The higher the coefficient of correlation between securities, the greater the reduction in the portfolio variance.
  - B. There is a linear relationship between the securities' coefficient of correlation and the portfolio variance.
  - C. The degree to which the portfolio variance is reduced depends on the degree of correlation between securities.
  - D. A and B.
  - E. A and C.

**The lower the correlation between the returns of the securities, the more portfolio risk is reduced.**

2. An investor who wishes to form a portfolio that lies to the right of the optimal risky portfolio on the Capital Allocation Line must:
  - A. lend some of her money at the risk-free rate and invest the remainder in the optimal risky portfolio.
  - B. borrow some money at the risk-free rate and invest in the optimal risky portfolio.
  - C. invest only in risky securities.
  - D. such a portfolio cannot be formed.
  - E. B and C

**The only way that an investor can create portfolios to the right of the Capital Allocation Line is to create a borrowing portfolio (buy stocks on margin). In this case, the investor will not hold any of the risk-free security, but will hold only risky securities.**

3. Suppose you held a well-diversified portfolio with a very large number of securities, and that the single index model holds. If the  $\sigma$  of your portfolio was 0.20 and  $\sigma_M$  was 0.16, the  $\beta$  of the portfolio would be approximately \_\_\_\_\_.
  - A. 0.64
  - B. 0.80
  - C. 1.25
  - D. 1.56
  - E. none of the above

**$s_p^2 / s_m^2 = b^2$ ;  $(0.2)^2 / (0.16)^2 = 1.56$ ;  $b = 1.25$ .**

4. An important difference between CAPM and APT is

- A. CAPM depends on risk-return dominance; APT depends on a no arbitrage condition.
- B. CAPM assumes many small changes are required to bring the market back to equilibrium; APT assumes a few large changes are required to bring the market back to equilibrium.
- C. implications for prices derived from CAPM arguments are stronger than prices derived from APT arguments.
- D. all of the above are true.
- E. both A and B are true.**

Under the risk-return dominance argument of CAPM, when an equilibrium price is violated many investors will make small portfolio changes, depending on their risk tolerance, until equilibrium is restored. Under the no-arbitrage argument of APT, each investor will take as large a position as possible so only a few investors must act to restore equilibrium. Implications derived from APT are much stronger than those derived from CAPM, making C an incorrect statement.

5. The risk-free rate is 7 percent. The expected market rate of return is 15 percent. If you expect a stock with a beta of 1.3 to offer a rate of return of 12 percent, you should

- A. buy the stock because it is overpriced.
- B. sell short the stock because it is overpriced.**
- C. sell the stock short because it is underpriced.
- D. buy the stock because it is underpriced.
- E. none of the above, as the stock is fairly priced.

$12\% < 7\% + 1.3(15\% - 7\%) = 17.40\%$ ; therefore, stock is overpriced and should be shorted.

6. A finding that \_\_\_\_\_ would provide evidence against the semistrong form of the efficient market theory.

- A. low P/E stocks tend to have positive abnormal returns
- B. trend analysis is worthless in determining stock prices
- C. one can consistently outperform the market by adopting a contrarian strategy based on price reversals
- D. A and B
- E. A and C**

Both A and C are inconsistent with the semistrong form of the EMH.

7. Consider the regression equation:

$$r_i - r_f = g_0 + g_1 b_i + g_2 s^2(e_i) + e_{it}$$

where:

$r_i - r_f$  = the average difference between the monthly return on stock  $i$  and the monthly risk-free rate

$b_i$  = the beta of stock  $i$

$s^2(e_i)$  = a measure of the nonsystematic variance of the stock  $i$

If you estimated this regression equation and the CAPM was valid, you would expect the estimated coefficient,  $g_1$  to be

A. 0

B. 1

C. equal to the risk-free rate of return.

D. equal to the average difference between the monthly return on the market portfolio and the monthly risk-free rate.

E. equal to the average monthly return on the market portfolio.

The variable measured by the coefficient  $g_1$  in this model is the market risk premium.

8. Each of two stocks, A and B, are expected to pay a dividend of \$5 in the upcoming year. The expected growth rate of dividends is 10% for both stocks. You require a rate of return of 11% on stock A and a return of 20% on stock B. The intrinsic value of stock A \_\_\_\_\_.

A. will be greater than the intrinsic value of stock B

B. will be the same as the intrinsic value of stock B

C. will be less than the intrinsic value of stock B

D. cannot be calculated without knowing the market rate of return.

E. none of the above is true.

$PV_0 = D_1/(k-g)$ ; given that dividends are equal, the stock with the larger required return will have the lower value.

9. Midwest Airline is expected to pay a dividend of \$7 in the coming year. Dividends are expected to grow at the rate of 15% per year. The risk-free rate of return is 6% and the expected return on the market portfolio is 14%. The stock of Midwest Airline has a beta of 3.00. The return you should require on the stock is \_\_\_\_\_.

A. 10%

B. 18%

C. 30%

D. 42%

E. none of the above

$6\% + 3(14\% - 6\%) = 30\%$ .

10. Suppose two portfolios have the same average return, the same standard deviation of returns, but portfolio A has a higher beta than portfolio B. According to the Sharpe measure, the performance of portfolio A \_\_\_\_\_.

- A. is better than the performance of portfolio B
- B. is the same as the performance of portfolio B
- C. is poorer than the performance of portfolio B
- D. cannot be measured as there is no data on the alpha of the portfolio
- E. none of the above is true.

The Sharpe index is a measure of average portfolio returns (in excess of the risk free return) per unit of total risk (as measured by standard deviation).

11. A Treasury bond due in one year has a yield of 5.7%; a Treasury bond due in 5 years has a yield of 6.2%. A bond issued by Ford Motor Company due in 5 years has a yield of 7.5%; a bond issued by Shell Oil due in one year has a yield of 6.5%. The default risk premiums on the bonds issued by Shell and Ford, respectively, are

- A. 1.0% and 1.2%
- B. 0.7% and 1.5%
- C. 1.2% and 1.0%
- D. 0.8% and 1.3%
- E. none of the above

Shell:  $6.5\% - 5.7\% = .8\%$ ; Ford:  $7.5\% - 6.2\% = 1.3\%$ .

12. You have just purchased a 10-year zero-coupon bond with a yield to maturity of 10% and a par value of \$1,000. What would your rate of return at the end of the year be if you sell the bond? Assume the yield to maturity on the bond is 11% at the time you sell.

- A. 10.00%
- B. 20.42%
- C. 13.8%
- D. 1.4%
- E. none of the above

$\$1,000/(1.10)^{10} = \$385.54$ ;  $\$1,000/(1.11)^9 = \$390.92$ ;  $(\$390.92 - \$385.54)/\$385.54 = 1.4\%$ .

13. Suppose that all investors expect that interest rates for the 4 years will be as follows:

Year	Forward Interest Rate
0	(today)5%
1	7%
2	9%
3	10%

What is the price of 3-year zero coupon bond with a par value of \$1,000?

- A. \$863.83
- B. \$816.58**
- C. \$772.18
- D. \$765.55
- E. none of the above

14. Holding other factors constant, the interest-rate risk of a coupon bond is higher when the bond's:

- A. term-to-maturity is lower.
- B. coupon rate is lower.**
- C. yield to maturity is higher.
- D. A and C
- E. none of the above.

The longer the maturity, the greater the interest-rate risk. The lower the coupon rate, the greater the interest-rate risk. The lower the yield to maturity, the greater the interest-rate risk. These concepts are reflected in the duration rules; duration is a measure of bond price sensitivity to interest rate changes (interest-rate risk).

15. Some of the problems with immunization are

- A. duration assumes that the yield curve is flat.
- B. duration assumes that if shifts in the yield curve occur, these shifts are parallel.
- C. immunization is valid for one interest rate change only.
- D. durations and horizon dates change by the same amounts with the passage of time.
- E. A, B, and C.**

Durations and horizon dates change with the passage of time, but not by the same amounts.

16. A covered call position is

- A. the simultaneous purchase of the call and the underlying asset.
- B. the purchase of a share of stock with a simultaneous sale of a put on that stock.
- C. the short sale of a share of stock with a simultaneous sale of a call on that stock.
- D.** the purchase of a share of stock with a simultaneous sale of a call on that stock.
- E. the simultaneous purchase of a call and sale of a put on the same stock.

Writing a covered call is a very safe strategy, as the writer owns the underlying stock. The only risk to the writer is that the stock will be called away, thus limiting the upside potential.

17. The Black-Scholes formula assumes that

- I) the risk-free interest rate is constant over the life of the option.
  - II) the stock price volatility is constant over the life of the option.
  - III) the expected rate of return on the stock is constant over the life of the option.
  - IV) there will be no sudden extreme jumps in stock prices.
- A. I and II
  - B. I and III
  - C. II and II
  - D.** I, II and IV
  - E. I, II, III, and IV

The risk-free rate and stock price volatility are assumed to be constant but the option value does not depend on the expected rate of return on the stock. The model also assumes that stock prices will not jump markedly.

18. Other things equal, the price of a stock call option is negatively correlated with the following factors

- A. the stock price.
- B. the time to expiration.
- C. the stock volatility.
- D.** the exercise price.
- E. A, B, and C.

The exercise price is negatively correlated with the call option price.

19. A hedge ratio for a call is always

- A. equal to one.
- B. greater than one.
- C.** between zero and one
- D. between minus one and zero.
- E. of no restricted value

Call option hedge ratios must be positive and less than 1.0, and put option ratios must be negative, with a smaller absolute value than 1.0.

20. Suppose that the risk-free rates in the United States and in the United Kingdom are 4% and 6%, respectively. The spot exchange rate between the dollar and the pound is \$1.60/BP. What should the futures price of the pound for a one-year contract be to prevent arbitrage opportunities, ignoring transactions costs.

- A. \$1.60/BP
- B. \$1.70/BP
- C. \$1.66/BP
- D. \$1.63/BP
- E. \$1.57/BP**

$$\text{\$1.60}(1.04/1.06) = \text{\$1.57/BP}.$$

## **PART 2 (OPEN QUESTIONS; 60 points at maximum)**

### **QUESTION 1. (15 points) Equilibrium pricing models**

#### **Part a. (3 points)**

Give the formula of the CAPM and explain its notation. What are the assumptions underlying the CAPM? How do they relate to empirical evidence?

The assumptions are:

- (a) The market is composed of many small investors, who are price-takers; i. e., perfect competition. In reality this assumption was fairly realistic until recent years when institutional investors increasingly began to influence the market with their large transactions.
- (b) All investors have the same holding period. Obviously, different investors have different goals, and thus have different holding periods.
- (c) Investments are limited to those that are publicly traded. In addition, it is assumed that investors may borrow or lend any amount at a fixed, risk-free rate. Obviously, investors may purchase assets that are not publicly traded; however, the dollar volume of publicly traded assets is considerable. The assumption that investors can borrow or lend any amount at a fixed, risk-free rate obviously is false. However, the model can be modified to incorporate different borrowing and lending rates.
- (d) Investors pay no taxes on returns and incur no transaction costs. Obviously, investors do pay taxes and do incur transaction costs.
- (e) All investors are mean-variance efficient. This assumption implies that all investors make decisions based on maximizing returns available at an acceptable risk level; most investors probably make decisions in this manner. However, some investors are pure wealth maximizers (regardless of the risk level); and other investors are so risk averse that avoiding risk is their only goal.
- (f) All investors have homogeneous expectations, meaning that given the same data all investors would process the data in the same manner, resulting in the same risk/return assessments for all investment alternatives.



**Part b. (5 points)**

Assume that the CAPM holds. The risk free rate  $r_f$  is 3% and the market return  $r_M$  is 5%. Consider the following two stocks:

Stocks	$\beta$	P/E
A	1.3	15.15
B	1.1	19.23

$\beta$  - market beta of each stock

P/E – price-earnings ratio

The retention ratio (**b**) for the two stocks is 0.5.

- i. Compute the return on equity (ROE) of the two stocks, assuming a constant growth model with endogenous earnings growth (round at the third decimal).

$$k_A = 0.03 + 1.3 \times (0.05 - 0.03) = 0.056, k_B = 0.03 + 1.1 \times (0.05 - 0.03) = 0.052$$

$$ROE_A = (15.15 \times 0.056 - 0.5) / (15.15 \times 0.5) = 0.046, ROE_B = (19.23 \times 0.052 - 0.5) / (19.23 \times 0.5) = 0.052$$

- ii. What value for **b** will make the P/E ratio the highest possible for stock A, if the ROE is kept at the level calculated in (i)?

**b** close to 0, as for stock A  $ROE < k$

- iii. What would the effect on the P/E ratio be if you change the retention ratio of stock B, keeping ROE constant? Why?

The P/E ratio will remain constant for changing **b**, as  $ROE = k$  for stock B.

**Part c. (5 points)**

Consider the Fama-French 3-factor model:

$$r_i = \alpha_i + \beta_i r_M + \gamma_i SMB + \delta_i HML + e_i$$

where  $r_i$  is the return of a stock  $i$ ,  $r_M$  is the market return, SMB is a factor that proxies for size, and HML – for value. The variances of the three factors are respectively  $\sigma_M^2$ ,  $\sigma_{SMB}^2$  and  $\sigma_{HML}^2$ .

The variance of the idiosyncratic source of risk is  $\sigma_e^2$  for all stocks  $i$ .

Assume that the idiosyncratic sources of risk are uncorrelated, and that the factors are uncorrelated as well.

Consider two stocks ( $i = \{1, 2\}$ ).

- i. Give an expression for the systematic risk of each of the two stocks.

$$\sigma_i^2 = \beta_i^2 \sigma_M^2 + \gamma_i^2 \sigma_{SMB}^2 + \delta_i^2 \sigma_{HML}^2$$

- ii. Construct an equally weighted portfolio of the two stocks. What is its non-systematic risk component? Compare it to the non-systematic risk component of the individual stocks.

$$\sigma_P^2 = \frac{1}{2} \sigma_e^2$$

- iii. Construct a portfolio out of the two stocks that has zero exposure to the size factor. Give an analytic expression of its weights.

The weights solve the following system of equations:

$$\begin{aligned} w_1 + w_2 &= 1 \\ \gamma_1 w_1 + \gamma_2 w_2 &= 0 \end{aligned}$$

With the solution:  $w_1 = -\frac{\gamma_2}{\gamma_1 - \gamma_2}$  and  $w_2 = 1 - \frac{\gamma_2}{\gamma_1 - \gamma_2}$ .

#### Part d. (2 points)

Consider the multifactor APT. There are two independent economic factors,  $F_1$  and  $F_2$ . The risk-free rate of return is 6%. The following information is available about two well-diversified portfolios:

Portfolio	$\beta$ on $F_1$	$\beta$ on $F_2$	Expected Return
A	1.0	2.0	19%
B	2.0	0.0	12%

Assume that no arbitrage opportunities exist. Compute the risk premium on factor  $F_1$  and the risk premium on factor  $F_2$ .

2A:  $38\% = 12\% + 2.0(RP_1) + 4.0(RP_2)$ ; B:  $12\% = 6\% + 2.0(RP_1) + 0.0(RP_2)$ ;  $26\% = 6\% + 4.0(RP_2)$ ;  $RP_2 = 5$ ;  
A:  $19\% = 6\% + RP_1 + 2.0(5)$ ;  $RP_1 = 3\%$ .

### QUESTION 2. (15 points) Portfolio construction and performance measurement

#### Part a. (6 points)

Consider a portfolio P that consists of a risky asset A and a risk-free asset with weights  $\omega$  and  $(1-\omega)$  respectively. Denote by  $r_A$  the return of the risky asset, and by  $r_f$  – the risk-free return. The standard deviation of the risky asset is  $\sigma_A$ .

- i. Give an analytic expression of the return of the portfolio ( $r_p$ ) and its variance ( $\sigma_p^2$ ). What is the expected return of the portfolio  $E[r_p]$ ?

$$\begin{aligned} r_p &= \omega r_A + (1 - \omega) r_f \\ \sigma_p^2 &= \omega^2 \sigma_A^2 \\ E[r_p] &= \omega E[r_A] + (1 - \omega) r_f \end{aligned}$$

- ii. Consider an investor who has the following utility function:

$$U = E[r_p] - \frac{1}{2} A \sigma_p^2$$

where A is the level of risk aversion of the investor. Obtain analytically the optimal weights in the risky asset  $\omega$  if the investor solves the following problem:

$$\max_{\omega} U = E[r_p] - \frac{1}{2} A \sigma_p^2$$

$$U = wE[r_A] + (1 - w)r_f - \frac{1}{2}Aw^2\sigma_A^2$$

Setting its first derivative with respect to w to zero:

$$U' = E[r_A] - r_f - Aw\sigma_A^2 = 0$$

We obtain for the optimal weight:

$$w^* = \frac{E[r_A] - r_f}{A\sigma_A^2}$$

- iii. Which value of A makes the investor indifferent between investing in the risky portfolio P and the risk-free asset? Give an analytic expression.

$$\begin{aligned} U(r_p) &= U(r_f) \\ E[r_p] - \frac{1}{2}A\sigma_p^2 &= r_f \\ A &= \frac{2(E[r_p] - r_f)}{\sigma_p^2} \end{aligned}$$

### Part b. (6 points)

Consider the following information about the performance of a fund and the market portfolio:

	Fund	Market portfolio
Average return	12%	9%
Standard deviation of returns	25%	20%
Beta	1.1	1

The risk free rate is 2%.

- Calculate the Sharpe ratio of the fund and of the market portfolio.  
Sharpe ratio (fund) =  $(0.12 - 0.02)/0.25 = 0.4$ ; Sharpe ratio (market) =  $(0.09 - 0.02)/0.2 = 0.35$
- Define the M2 performance measure, also giving a graphic interpretation. Hint: recall that it involves creating a hypothetical portfolio made up of a combination of the fund portfolio and a risk-free investment. What is the link between the M2 measure and the Sharpe ratio?  
The M2 measure gives the excess return of a hypothetical portfolio over the market. The hypothetical portfolio is a linear combination of the risk-free asset and the managed portfolio, and has the same volatility as the market portfolio.
- Calculate the weights of the hypothetical portfolio used to calculate the M2 measure. Obtain its return.  
Weight(risk-free) =  $1 - 0.2/0.25 = 0.2$ ; weight(managed portfolio) =  $1 - 0.2 = 0.8$ .  
Return(hypothetic portfolio) =  $0.8 * 0.12 + 0.2 * 0.09 = 0.1$
- Calculate the M2 measure for the fund.  
M2 =  $0.1 - 0.09 = 0.09$

### Part c. (3 points)

You are evaluating the market timing ability of a portfolio manager. In order to do so, you use the following regression equation:

$$r_P - r_f = a + b(r_M - r_f) + c(r_M - r_f)1_{r_M > r_f} + e_P$$

where  $r_p$  is the return of the portfolio,  $r_f$  is the risk-free rate,  $r_M$  is the market return,  $1_{r_M > r_f}$  is an indicator function equal to 1 if the market return is higher than the risk free rate and 0 otherwise, and  $e_p$  is the error term.

- i. How would you test for the timing ability of the portfolio manager, using the above regression equation? Give also a graphic interpretation.  
Test for  $c > 0$  which would indicate market timing ability. Graph in lecture 6.
- ii. How would you define an ideal situation of perfect foresight (also give a graphic interpretation)? Using this, how could you compute the value of market timing?  
Perfect foresight is equivalent to holding a call option on the index portfolio. Graph in lecture 6.

### QUESTION 3. (15 points) Fixed Income

#### Part a. (3 points)

Discuss the meaning, usage, similarities and differences between the following 2 couples of concepts:

- i. duration of a bond and delta of an option  
Duration measures the interest rate sensitivity of the bond price. The option delta measures the stock price sensitivity of the option. Both are linear approximations. The delta of the option gives the fractions of stocks needed to build a hedge portfolio; while this is not the case with duration (explicit duration matching is needed).
- ii. convexity of a bond and gamma of an option  
Convexity measures the second order effect of interest rate changes on the bond price, while the gamma measures the second order effect of stock price changes on the option price.

#### Part b. (5 points)

Consider the following forward rates:  $f_1 = 3\%$ ,  $f_2 = 4\%$ , and  $f_3 = 4.5\%$ .

- i. Express the prices of zero-coupon bonds with maturities of 1, 2 and 3 years using the forward rates. Solve for them.  
 $P_1 = 100/(1+f_1) = 97.09$ ,  $P_2 = 100/((1+f_1)*(1+f_2)) = 93.35$ ,  $P_3 = 100/((1+f_1)*(1+f_2)*(1+f_3)) = 89.33$ .
- ii. Find the pure yield curve from the forward curve.  
 $z_1 = f_1$   
 $(1+z_2)^2 = (1+f_1)*(1+f_2)$   
 $(1+z_3)^3 = (1+f_1)*(1+f_2)*(1+f_3)$   
Solving this system yields:  $z_1 = 0.03$ ,  $z_2 = 0.035$ ,  $z_3 = 0.038$ .
- iii. You are offered to enter in the following agreement: lend \$10000 at the start of year 3 at 5% for 1 year. What is the value of the agreement today?  
 $-10000/(1+z_2)^2 + 10000*(1+0.05)/(1+z_3)^2 = \$44.67$ .
- iv. What are the cash-flows of the agreement (now (beginning of year 1), at the end of year 1, 2 and 3)? How can you hedge this agreement today? (Hint: use zero-coupon bonds) Give the portfolio for the hedging strategy and its cash-flows.

Cash-flows of the agreement:

now	44.67
1	0
2	-10000
3	10500

Hedging the agreement today:

now Long 100 2-year zero bonds, short 105 3-year zero bonds,  
value of the position:  $100 \cdot 93.35 - 105 \cdot 98.33 = -44.67$

1	0
2	$-100 \cdot 100$
3	$105 \cdot 100$

**Part c. (4 points)**

You hold a bond portfolio with a duration  $D$  and convexity  $C$ . The current zero curve is flat at  $y\%$ . Construct a portfolio of zero coupon bonds with maturities of 1, 2 and 3 years that match the duration and the convexity of your portfolio. Solve analytically for the weights of the new zero coupon bond portfolio.

Denote by  $a$ ,  $b$ , and  $c$  the weights of the 3 zero coupon bonds in the portfolio. They solve the following system:

$$a + b + c = 1$$

$$a + 2b + 3c = D$$

$$2a/(1+y)^2 + 6b/(1+y)^2 + 12c/(1+y)^2 = C$$

Solving it yields:

$$a = 3 - 3D + 1/2C(1+y)^2$$

$$b = 5D - 3 - C(1+y)^2$$

$$c = 1/2C(1+y)^2 - 2D + 1.$$

**Part d. (3 points)**

Consider the data on the following two coupon bonds:

Bond	Maturity	Coupon	Yield	Face value
A	2	7%	2%	100
B	3	4%	3%	100

- i. Compute the duration and the modified duration of the two bonds.

The prices of the two bonds:

$$P_A = 7/1.02 + 107/1.02^2 = 109.71$$

$$P_B = 4/1.03 + 4/1.03^2 + 104/1.03^3 = 102.83$$

Duration:

$$D_A = 1 \cdot 7/1.02/109.71 + 2 \cdot 107/1.02^2/109.71 = 1.94$$

$$D_B = 1 \cdot 4/1.03/102.83 + 2 \cdot 4/1.03^2/102.83 + 3 \cdot 104/1.03^3/102.83 = 2.89$$

Modified duration:

$$D_A^* = 1.94/(1+0.02) = 1.90$$

$$D_B^* = 2.89/(1+0.03) = 2.80$$

- ii. You have a portfolio consisting of a long position in 3 bonds of type B and a short position in 2 bonds of type A. Calculate the duration and the modified duration of the portfolio.

$$\text{Value of the portfolio: } -2 \cdot 109.71 + 3 \cdot 102.83 = 89.07$$

$$\text{Weights of the portfolio: } w_A = -2 \cdot 109.71/89.07 = -2.46, w_B = 3 \cdot 102.83/89.07 = 3.46$$

$$\text{Portfolio modified duration: } D^* = -2.46 \cdot 1.90 + 3.46 \cdot 2.80 = 5.03.$$

- iii. Using duration approximation, what is the change in the value of the portfolio if the yield curve shifts upwards by 100 basis points? And if it shifts upwards by 10 basis points? In which of the two cases the approximation will be more exact and why?

100bp:  $\Delta P/P = -5.03 \times 0.01 = -0.5$

10bp:  $\Delta P/P = -0.005$  (more exact as smaller yield change).

#### QUESTION 4. (15 points) Option pricing

##### Part a. (4 points)

Consider an Asian call option. It pays the maximum between zero and the difference between the average realized price of the underlying during the life of the option (denote it by  $M$ ), and the strike price ( $K$ ). The price of the stock today is  $S_0 = \$10$ . Its future evolution can be modeled by a binomial tree, where at each period it can go up by 15% or down by 10%. The strike price is  $K = \$9.5$ . The periodic risk-free rate is 0.5%. The stock pays no dividends and the option can be exercised only at maturity.

Build a 2-period binomial tree (so for  $t=0, t=1, t=2$ ) and calculate the price of the option based on it.

The binomial price tree:  $S_0 = 10$ ,  $S_u = 10 \times 1.15 = 11.5$ ,  $S_d = 10 \times 0.9 = 9$ ,  $S_{uu} = 13.225$ ,  $S_{ud} = S_{du} = 10.35$ ,  $S_{dd} = 8.1$ .

The pay-off at maturity ( $C$ ):  $C_{uu} = \max(0, (10, 11.5, 13.225)/3 - 9.5) = 2.075$ ;  $C_{ud} = \max(0, (10, 11.5, 10.35)/3 - 9.5) = 1.12$ ;  $C_{du} = \max(0, (10, 9, 10.35)/3 - 9.5) = 0.28$ ;  $C_{dd} = \max(0, (10, 9, 8.1)/3 - 9.5) = 0$ .

The risk-neutral probability  $Q = ((10 \times (1 + 0.005) - 9) / (11.5 - 9)) = 0.42$ .

The call price:  $C_u = (0.42 \times 2.075 + (1 - 0.42) \times 1.12) / (1 + 0.005) = 1.51$ ;  $C_d = (0.42 \times 0.28 + (1 - 0.42) \times 0) / (1 + 0.005) = 0.12$ ;  $C = (0.42 \times 1.51 + (1 - 0.42) \times 0.12) / (1 + 0.005) = 0.7$ .

##### Part b. (7 points)

You are evaluating a stock that is currently selling for \$30 per share. Over the investment period of 1 year you think that the stock price might get as low as \$25 or as high as \$40. There is a call option available on the stock with an exercise price of \$35. The annual interest rate is 6%. Answer the following questions about hedging your position in the stock. Assume that you will buy one share.

- What is the hedge ratio (Hint: use the option pay-offs and the stock prices for the high and the low scenarios)?  
 $(5 - 0) / (40 - 25) = 1/3$ . [If the stock price ends at \$40 the call is worth \$5; if it ends at \$25 the call is worth \$0.]
- How much would you borrow to purchase the stock? (Hint: make sure the value of your stock portfolio at the end of the holding period is non-negative)  
Borrow the present value of the anticipated minimum stock price =  $\$25 / 1.06 = \$23.58$
- What is the amount of your net investment in the stock?  
 $\$30 - 23.58 = \$6.42$

- iv. Complete the table below to show the value of your stock portfolio at the end of the holding period.

Scenario	Low Stock Price	High Stock Price
Value of Stock at Year End	\$25	\$40
Repayment of Loan	-\$25	-\$25
Total	\$0	\$15

- v. How many call options will you combine with the stock to construct the perfect hedge? Will you buy the calls or sell the calls?

Since the hedge ratio is  $1/3$  buy one stock and sell three call options.

- vi. Show the option values in the table below.

Scenario	Low Stock Price	High Stock Price
Value of Call Position	\$0	\$15 $\{= 3 * \$5\}$

- vii. Show the net payoff to your portfolio in the table below.

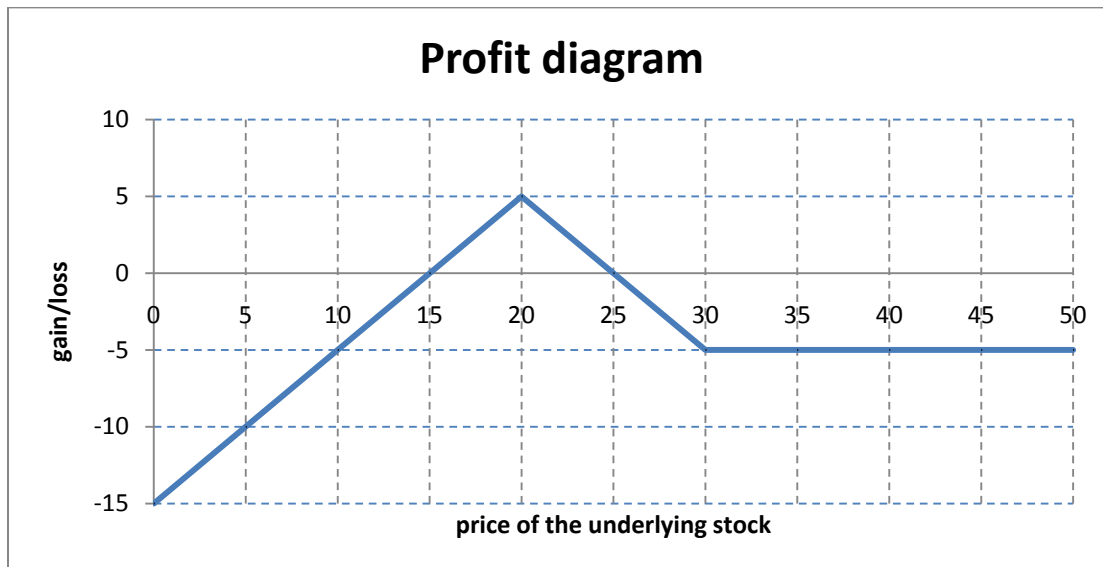
Scenario	Low Stock Price	High Stock Price
Value of Stock at Year End	\$25	\$40
Value of Call Position	-\$0	-\$15
Total	\$25	\$25

- viii. What must the price of one call option be?

The value of the stock portfolio equals the value of three calls. The net investment in the stock portfolio is \$6.42 so this must equal the value of the three calls.  $\$6.42 = 3C$ , and  $C = \$2.14$ .

**Part c. (4 points)**

Consider the profit diagram of a *put ratio vertical spread* illustrated below. What is the premium paid/received for the position? Replicate the profit diagram using options. Pay attention to the scale of the horizontal and the vertical axes. If the stock price today is \$21, comment on the moneyness of the options used in the strategy. When would you use such a strategy (comment in terms of the volatility and the direction of the market)?



Short 2 puts @ 20 (out of the money)

Long 1 put @ 30 (in the money)

Premium paid for the position is 5

Limited loss on the downside, limited gain on the upside (equal to the difference between the two strike prices less the premium paid for the position). Used when the investor is neutral as to the market direction but is expecting low volatility.