vrije Universiteit amsterdam

Studentnumber:
Name:

School of Business and Economics

Exam:	Introduction to Time Series and Dynamic Econometrics
Code:	E_MFAE_ITSDE
Examinator:	Karim Moussa
Co-reader:	Mariia Artemova
Date:	December 11, 2023
Time:	15:30
Duration:	2 hours
Calculator allowed:	**Yes**
Graphical calculator allowed:	**No**
Scrap paper allowed	**Yes**
MC forms allowed	**No**
Number of questions:	3 (with sub-questions)
Type of questions:	Open and multiple choice
Answer in:	English

Remarks:
- Start each question (MC/Q1/Q2) on a separate page.
- MC questions: unless explicitly stated otherwise, select one answer. Write down only the answers (the rest is not graded); you can put your derivations on scrap paper.
- For the open questions: provide a justification for each answer.
- If you hand in multiple sets of papers, write your name and student number on of each of them.
- You are **not** allowed to take any paper with you, neither the exam, nor the scrap paper.

Credit score:	100 credits counts for a 10
Grades:	The grades will be made public on: November 8
Inspection:	TBD
Number of pages:	6 (including front page)

Good luck!

Multiple choice part [40 points]

1. **(5 pts)** Let $\{u_t\} \sim WN(0, \sigma^2)$ with $\sigma^2 = 1$. Which of the following time series are weakly stationary? Select <u>ALL</u> the correct statements.

   (a) $X_t = c$, where $c \neq 0$ is constant

   (b) $X_t = -X_{t-1} + u_t$

   (c) $X_t = 0.7X_{t-1} + 0.3X_{t-3} + u_t$

   (d) $X_t = 0.7u_t + 0.3u_{t-1}$

   (e) $X_t = \cos(t) + u_t$.

2. **(5 pts)** Consider the process

$$X_t = \frac{(1-0.6L)^2}{(1+0.5L)} u_t, \qquad \{u_t\} \sim WN(0, \sigma^2).$$

   The process $\{X_t\}$ is of the following type:

   (a) ARMA(1,1)

   (b) ARMA(1,2)

   (c) ARMA(2,1)

   (d) ARMA(2,2)

3. **(5 pts)** Consider the stable AR(1) model:

$$X_t = \phi_1 X_{t-1} + u_t, \qquad \{u_t\} \sim IID(0, \sigma^2),$$

   where the errors are normally distributed, $u_t \sim N(0, \sigma^2)$. Assume that we have two observations, that is $T = 2$, $X_1 = 5$ and $X_2 = 2$. What is the conditional maximum likelihood estimate of ϕ_1?

   <u>Remark</u>: Recall that the density of the univariate normal distribution with mean μ and variance V is given by

$$f(x) = \frac{1}{\sqrt{2\pi V}} \exp\left[-\frac{(x-\mu)^2}{2V}\right].$$

   (a) $\hat{\phi}_1 = 0.2$

   (b) $\hat{\phi}_1 = 0.4$

   (c) $\hat{\phi}_1 = 0.6$

   (d) $\hat{\phi}_1 = 0.8$

   (e) $\hat{\phi}_1 = 1.0$

   (f) $\hat{\phi}_1 = 1.2$

4. **(5 pts)** Consider the ARMA(1,2) model

$$X_t = \phi_1 X_{t-1} + \varepsilon_t + \theta_2 \varepsilon_{t-2}, \qquad \{\varepsilon_t\} \sim IID(0, \sigma^2),$$

   with coefficients $\phi_1 = 0.8$, $\theta_2 = 0.7$, and $\sigma^2 = 1$. Suppose you observe data $D_T = \{X_1, \ldots, X_T\}$ for some $T > 1$, and assume that the errors ε_1 and ε_0 are given. What is the <u>variance</u> of the 2-step ahead forecast error $e_{T+2} = X_{T+2} - \mathbb{E}[X_{T+2}|D_T]$?

(a) 0.64

(b) 0.92

(c) 1.28

(d) 1.64

5. (**5 pts**) Consider the following ADL(1,4) model,

$$Y_t = \alpha + \phi_1 Y_{t-1} + \beta_0 X_t + \beta_4 X_{t-4} + \varepsilon_t, \qquad \{\varepsilon_t\} \sim IID(0, \sigma^2),$$

with parameters $\alpha = 2$, $\phi_1 = 0.5$, $\beta_0 = 4$, $\beta_4 = -1$, and with $\{X_t\}$ a weakly stationary and exogenous process. What is the long-run equilibrium value of Y_t for $\overline{X} = 1.5$?

(a) 11

(b) 12

(c) 13

(d) 14

6. (**5 pts**) Consider the AR(4) process

$$X_t = \phi_1 X_{t-1} + \phi_2 X_{t-2} + \phi_4 X_{t-4} + u_t, \qquad \{u_t\} \sim IID(0, 1).$$

Suppose the origin is $x = 0$, and the parameters are $\phi_1 = 0.8$, $\phi_2 = 0.3$, and $\phi_4 = -0.5$. What is the IRF

$$\widetilde{X}_t = x + \partial X_t / \partial u_s \cdot \epsilon$$

for time $t = s + 2$ with shock size $\epsilon = 2$?

(a) 1.64

(b) 1.74

(c) 1.88

(d) 1.96

7. (**5 pts**) Consider the following stochastic process:

$$X_t = \alpha + \phi X_{t-1} + \delta t + u_t, \qquad \{u_t\} \sim \text{WN}(0, \sigma^2).$$

Which restrictions on the parameters α, δ, and ϕ should be imposed in order for a process X_t to be a random walk with drift?

(a) $\alpha = 0,\ \delta = 0,\ \phi = 1$

(b) $\alpha = 0,\ \delta \neq 0,\ \phi = 1$

(c) $\alpha = 0,\ \delta = 0,\ \phi \neq 1$

(d) $\alpha \neq 0,\ \delta = 0,\ \phi = 1$

8. (**5 pts**) Consider the following ADF regression for an AR(3) model:

$$\Delta X_t = \underset{(0.05)}{-0.138} X_{t-1} + \underset{(0.07)}{0.87}\,\Delta X_{t-1} - \underset{(0.08)}{0.15}\,\Delta X_{t-2} + u_t, \quad u_t \sim \text{WN}(0, \sigma^2),$$

If you would perform an ADF test based on these estimation results (standard errors between parentheses) and a 5% significance level, what would be your conclusion?

DF Critical Values

Significance level	0.01	0.05	0.10
		Standard	
Critical Value	-2.58	-1.95	-1.62
		Intercept	
Critical Value	-3.43	-2.86	-2.57
		Intercept + Trend	
Critical Value	-3.96	-3.41	-3.13

(a) $X_t \sim I(-1)$

(b) $X_t \sim I(0)$

(c) $X_t \sim I(1)$

(d) $X_t \sim I(d)$ with $d \geq 1$

Open Question 1: ARMA models [40 points]

In this question we consider the MA(q) model defined by

$$X_t = \alpha + \varepsilon_t + \sum_{j=1}^{q} \theta_j \varepsilon_{t-j}, \qquad \{\varepsilon_t\} \sim WN(0, \sigma^2),$$

with initial coefficient $\theta_0 = 1$ and finite error variance $\sigma^2 > 0$.

(a) (**2pts**) Write the general MA(q) model above in lag polynomial form, and give an expression for the lag polynomial.

(b) (**5pts**) Suppose that $\alpha = 0$ and $q = 1$, which yields the MA(1) model without intercept. State the condition under which the errors have an AR(∞) representation,

$$\varepsilon_t = \sum_{j=0}^{\infty} \pi_j X_{t-j}.$$

Give the value of the coefficient π_3.

(c) (**15pts**) Consider the MA(3) model

$$X_t = \alpha + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_3 \varepsilon_{t-3}, \qquad \{\varepsilon_t\} \sim IID(0, \sigma^2). \qquad (1)$$

Derive the unconditional mean $\mathbb{E}[X_t]$, variance $\mathbb{V}ar[X_t]$, and the autocovariance function $\gamma(h) = \mathbb{C}ov[X_t, X_{t-h}]$ for all values $h \in \mathbb{Z}$.

(d) (**5pts**) For the MA(3) model in Equation (1), derive the conditional mean and variance

$$\mathbb{E}[X_t | D_{t-1}] \qquad \text{and} \qquad \mathbb{V}ar[X_t | D_{t-1}], \qquad t > 1,$$

where $D_{t-1} = \{X_1, \ldots, X_{t-1}\}$ and we assume that $\varepsilon_0, \varepsilon_{-1}, \varepsilon_{-2}$ are given.

(e) (**7 pts**) For the MA(3) model in Equation (1), assume that the errors follow the standard normal distribution, $\varepsilon_t \sim N(0, \sigma^2)$, in addition to being independent.

- Derive the log likelihood based on the prediction error decomposition, assuming that $\varepsilon_0, \varepsilon_{-1}, \varepsilon_{-2}$ are given.
- Discuss the advantages of this approach over the log likelihood based on joint normality of the errors.

Remark: The density of the univariate normal distribution can be found on p.2.

(f) (**6 pts**) State the part of the general weak stationarity theorem that can be used to show that a process is weakly stationary (state **only** this part of the theorem). Use it to show that the MA(q) process is weakly stationary.

Open Question 2: Unit-Root Non-Stationarity and Cointegration [20 points]

(a) (**2 pts**) When do we say that a time series is integrated of order d?

(b) (**5 pts**) Suppose that X_t is an $I(1)$ process and $Y_t = X_t + \varepsilon_t$ with $\{\varepsilon_t\} \sim IID(0, 1)$.

- What can be said about the order of integration of Y_t?
- Are X_t and Y_t cointegrated? Explain.

(c) (**3 pts**) Rewrite the process $Y_t = X_t + \varepsilon_t$ in the error correction form.

(d) (**5 pts**) In practice it is usually unknown whether two time series $\{X_t\}$ and $\{Y_t\}$ are cointegrated. Describe how you would test for cointegration given data on these two time series.

(e) (**5 pts**) Suppose that after testing you conclude that the time series $\{X_t\}$ and $\{Y_t\}$ are cointegrated. Explain which model you would estimate and why, and describe how you would estimate it.