vrije Universiteit amsterdam

# Preparation Exam
# Introduction to Time Series

Bachelor Econometrics and Operations Research
Faculty of Economics and Business Administration

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| Exam: | Introduction to Time Series |
| Code: | E_EOR3_ITS |
| Coordinator: | dr. F. Blasques |
| Date: | – |
| Time: | – |
| Duration: | 2 hours and 45 minutes |
| | |
| Calculator: | Not allowed |
| Graphical calculator: | Not allowed |
| Number of questions: | 4 |
| Type of questions: | Open |
| Answer in: | English |
| | |
| Credit score: | 100 credits counts for a 10 |
| Grades: | Made public within 10 working days |
| Inspection: | By appointment (send e-mail to f.blasques@vu.nl) |
| Number of pages: | 3, including front page |

- Read the entire exam carefully before you start answering the questions.
- Be clear and concise in your statements, but justify every step in your derivations.
- The questions should be handed back at the end of the exam. Do not take it home.

**Good luck!**

**Question 1 [35 points]  ARMA Models**

Let $\{X_t\}_{t\in\mathbb{Z}}$ be a time-series generated by an ARMA$(2,2)$ model

$$X_t = \phi_1 X_{t-1} + \phi_2 X_{t-2} + \epsilon_t + \theta_1 \epsilon_{t-1} + \theta_2 \epsilon_{t-2} \quad , \quad t \in \mathbb{Z} \ ,$$

where $\{\epsilon_t\}_{t\in\mathbb{Z}}$ is a sequence of white noise innovations with variance $\sigma_\epsilon^2 > 0$.

(a) Give the definitions of strict stationarity and weak stationarity. Can you give an example of a strictly stationary time-series that is not weakly stationary? Justify your answer.

(b) Please rewrite the ARMA$(2,2)$ model in lag polynomial form $\phi(L)X_t = \theta(L)\epsilon_t$. Give an expression for the polynomials $\phi(L)$ and $\theta(L)$. Use the general weak stationarity theorem to show that $\{X_t\}_{t\in\mathbb{Z}}$ is weakly stationary if $\phi(L)$ is invertible.

(c) Suppose that $|\phi_1| < 1$, $\theta_1 \neq 0$ and $\phi_2 = \theta_2 = 0$. Calculate the unconditional mean and variance of $\{X_t\}_{t\in\mathbb{Z}}$. In other words, derive an expression for $\mathbb{E}(X_t)$ and $\mathbb{V}\text{ar}(X_t)$.

(d) Suppose now that $\phi_1 \neq 0$ and $\phi_2 \neq 0$ and $\theta_1 = \theta_2 = 0$. Additionally, assume that the innovations $\{\epsilon_t\}_{t\in\mathbb{Z}}$ are independent and identically distributed (iid) Gaussian random variables $\{\epsilon_t\}_{t\in\mathbb{Z}} \sim \text{NID}(0,\sigma_\epsilon^2)$. Produce the 2-step ahead forecast and the variance of the 2-step ahead forecast error. Derive 95% confidence bounds for your forecast.

**Question 2 [15 points]  ML Estimation**

Let $\{X_t\}_{t\in\mathbb{Z}}$ be a time-series generated by a MA(1) process,

$$X_t = \theta_1 \epsilon_{t-1} + \epsilon_t \quad , \quad t \in \mathbb{Z} \ ,$$

with independent and identical Gaussian innovations $\{\epsilon_t\}_{t\in\mathbb{Z}} \sim \text{NID}(0,\sigma_\epsilon^2)$ and $\sigma_\epsilon^2 > 0$. Note that this implies that $\epsilon_t$ has the following probability density function:

$$f(\epsilon_t;\sigma_\epsilon^2) = \frac{1}{\sqrt{2\pi\sigma_\epsilon^2}} e^{-\epsilon_t^2/2\sigma_\epsilon^2} \quad , \quad t \in \mathbb{Z}$$

(a) Give an expression of the log likelihood function for the unknown parameters $(\theta_1, \sigma_\epsilon^2)$ using the joint Gaussianity of the sample $X_1, ..., X_T$.

(b) Write down the conditional likelihood function using prediction error decomposition starting at $t = 2$.

**Question 3 [15 points] Unit-Root Testing**

Let $\{X_t\}_{t\in\mathbb{Z}}$ be a time-series generated by an AR(2) process,

$$X_t = \phi_1 X_{t-1} + \phi_2 X_{t-2} + \epsilon_t \quad , \quad t \in \mathbb{Z} \ ,$$

where $\{\epsilon_t\}_{t\in\mathbb{Z}} \sim \text{WN}(0, \sigma_\epsilon^2)$ with $\sigma_\epsilon^2 > 0$ and consider the ADF regression,

$$\Delta X_t = \beta X_{t-1} - \phi_2 \Delta X_{t-1} + \epsilon_t.$$

(a) (**7pts**) Re-write the AR(2) model in the ADF regression form. Show that testing the hypothesis $\text{H}_0 : \beta = 0$ is equivalent to testing for a unit root in the polynomial $\phi(z) = 1 - \phi_1 z - \phi_2 z^2$.

(b) Why is it important to use a general-to-specific approach in the specification of the ADF regression? Justify your answer.

**Question 4 [35 points] ADL, Error Correction and Cointegration**

Let $\{Y_t\}_{t\in\mathbb{Z}}$ be a time-series generated by an ADL$(1,1)$ process,

$$Y_t = \alpha + \phi Y_{t-1} + \beta_0 X_t + \beta_1 X_{t-1} + \epsilon_t \quad , \quad t \in \mathbb{Z} \ ,$$

where $\{\epsilon_t\}_{t\in\mathbb{Z}}$ is a sequence of iid white noise innovations with variance $\sigma_\epsilon^2 > 0$.

(a) Suppose $|\phi| < 1$. Use the ADL(1,1) model to derive the $h$-step ahead multiplier for $h = 1, 2, 3$. Derive the long run multiplier and explain its meaning.

(b) Suppose that $\{X_t\}_{t\in\mathbb{Z}}$ is generated by the following AR(2) model with intercept

$$X_t = \gamma_0 + \gamma_1 X_{t-1} + \gamma_2 X_{t-2} + u_t \quad , \quad t \in \mathbb{Z} \ ,$$

where $\{u_t\}_{t\in\mathbb{Z}}$ is a sequence of iid white noise innovations with variance $\sigma_u^2 > 0$. Calculate the the impulse response function (IRF) of $\{Y_t\}_{t\in\mathbb{Z}}$ given the origin $x$ for the time-series $\{X_t\}_{t\in\mathbb{Z}}$, the origin $y$ for the time-series $\{Y_t\}_{t\in\mathbb{Z}}$, and a shock of magnitude $v$ in the innovation $\{u_t\}_{t\in\mathbb{Z}}$ at time $t = s$. In particular, give the IRF for $t = s - 1$, $t = s$, $t = s + 1$ and $t = s + 2$.

(c) Let $\{Y_t\}_{t\in\mathbb{Z}}$ and $\{X_t\}_{t\in\mathbb{Z}}$ be $I(1)$ time series. Suppose that you have obtained the following estimates for the parameters of the ADL(1,1) model above:

| Parameter | $\alpha$ | $\phi$ | $\beta_0$ | $\beta_1$ | $\sigma_\epsilon^2$ |
|---|---|---|---|---|---|
| Estimate | 0.11 | 0.94 | 1.28 | 0.01 | 1.14 |

Furthermore, suppose that the $p$-values you obtained indicate that all parameters are significantly different from zero at the 5% significance level. Does $\{X_t\}_{t\in\mathbb{Z}}$ Granger cause $\{Y_t\}_{t\in\mathbb{Z}}$? Can $\{Y_t\}_{t\in\mathbb{Z}}$ Granger cause $\{X_t\}_{t\in\mathbb{Z}}$? Are $\{Y_t\}_{t\in\mathbb{Z}}$ and $\{X_t\}_{t\in\mathbb{Z}}$ cointegrated? Justify your answers carefully and in detail.