Exam

Question 1

20 pts

Consider the following ARCH(2) model

$$y_t = \sigma_t \varepsilon_t, \quad \varepsilon_t \sim N(0,1),$$
$$\sigma_t^2 = 0.1 + 0.5 y_{t-1}^2 + 0.3 y_{t-2}^2.$$

The observed log-returns at time T and $T+1$ are $y_T = 1.0$ and $y_{T-1} = -2.0$.

Answer the following questions:

1. Compute the conditional variance at time $T+1$ (i.e. obtain σ_{T+1}^2).
2. Compute the conditional probability that y_{T+1} is lower than -1.0 (i.e. obtain $Pr(y_{T+1} < -0.1 | Y^T)$). *Note: the result can be expressed in terms of the standard normal cdf* $\Phi(\cdot)$.
3. Would you expect the probability $Pr(y_{T+1} < -0.1 | Y^T)$ to become larger or smaller if $y_T = 1.0$ is replaced by $y_T = -1.5$? Justify your answer. Do not compute the probability.

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Question 2

10 pts

Given is the following R code with a for loop for the filtered conditional volatility to estimate an ARCH(2) model by ML.

```
omega <- exp(par[1])

alpha1 <- exp(par[2])/(1+exp(par[2]))

alpha2 <- exp(par[3])/(1+exp(par[3]))

sig2 <- rep(var(x),n)

for(t in 2:n){

  sig2[t] <- omega + alpha1 * x[t]^2 + alpha2 * x[t-1]^2

}
```

where **x** is a vector of length **n** that contains the observed log-returns.

Is the code correct or is there something wrong? How would you adjust the above code to obtain the filtered volatility of an ARCH(3) model instead of ARCH(2)? Your answer may contain the lines of code that you would use.

Question 3

25 pts

Consider the following scalar DVECH(0,1) model for a bivariate vector $\boldsymbol{y}_t = (y_{1,t}, y_{2,t})^\top$ of stock log-returns:

$$\boldsymbol{y}_t = \boldsymbol{\Sigma}_t^{1/2} \boldsymbol{\varepsilon}_t \ , \quad \{\boldsymbol{\varepsilon}_t\}_{t \in \mathbb{Z}} \sim \text{NID}_2(\boldsymbol{0}_2, \boldsymbol{I}_2) \ ,$$

where the conditional covariance matrix $\boldsymbol{\Sigma}_t$ is given by

$$\begin{bmatrix} \sigma_{1,t}^2 & \sigma_{21,t} \\ \sigma_{21,t} & \sigma_{2,t}^2 \end{bmatrix} = \begin{bmatrix} 1 & 0.5 \\ 0.5 & 2 \end{bmatrix} + 0.5 \begin{bmatrix} y_{1,t-1}^2 & y_{1,t-1} y_{2,t-1} \\ y_{1,t-1} y_{2,t-1} & y_{2,t-1}^2 \end{bmatrix},$$

where the observed log-returns at time T are $y_{1,T} = -2.0$ and $y_{2,T} = 1.0$.

Answer the following questions:

1. Calculate the conditional 1%-level VaR at time $T+1$ of the following portfolio $y_{p,t} = 0.5 y_{1,t} + 0.5 y_{2,t}$ (the quantile of level 0.01 of a standard normal is $z_{0.01} = -2.33$).
2. Calculate the unconditional variance of the portfolio $\mathbb{V}ar(y_{p,t})$.
3. A colleague of yours claims the following: *"When we estimate a scalar DVECH(1,1) model using covariance targeting we only have 2 parameters to optimize in the likelihood irrespective of the number of assets n. Therefore, we can use covariance targeting even when the number of assets n is larger than the sample size T".* Do you agree with the claim? Justify your answer.

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Question 4

15 pts

A colleague of yours makes the following 2 statements:

1. *"I have estimated a GARCH(1,1) and a GARCH(2,2) using a series of log-returns. I have obtained that the AIC and BIC of the of the GARCH(1,1) are larger than the AIC and BIC of the GARCH(2,2). This suggests that the GARCH(1,1) is misspecified."*
2. *"In GARCH models, I can test the assumption of normal distribution of the error ε_t using the Jarque Bera test on the log-returns."*

Do you agree with the above statements? Justify your answers.

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Question 5

20 pts

Consider the following parameter-driven regression model with an exponential link function for β_t

$$y_t = \beta_t x_t + \varepsilon_t, \quad \beta_t = \exp(f_t)$$

$$f_t = \eta_t + \phi_1 \eta_{t-1} + \phi_2 \eta_{t-2},$$

where $\{\varepsilon_t\}_{t\in\mathbb{Z}}$, $\{\eta_t\}_{t\in\mathbb{Z}}$ and $\{x_t\}_{t\in\mathbb{Z}}$ are $NID(0,1)$ sequences independent of each other.

Recall that if z_t is a *normal* random variable $z_t \sim N(\mu, \sigma^2)$, then $\exp(z_t)$ has a *log-normal* distribution, which is denoted $\exp(z_t) \sim \log\text{-}N(\mu, \sigma^2)$, and furthermore, has mean given by

$$\mathbb{E}[\exp(z_t)] = \exp(\mu + \sigma^2/2).$$

Use this information to answer the following questions:

1. Obtain the autocovariance function of y_t, (i.e. obtain $\mathbb{C}ov(y_t, y_{t-k})$ for $k \geq 1$). Is y_t independent of y_{t-k}?
2. Find the conditional variance of y_t given x_t (i.e. obtain $\mathbb{V}\mathrm{ar}(y_t|x_t)$).

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Question 6

10 pts

Consider the following SV model

$$y_t = \sigma_t \epsilon_t, \quad \sigma_t^2 = \exp(f_t),$$

$$f_t = \omega + \beta f_{t-1} + \eta_t,$$

where $\{\epsilon_t\}_{t\in\mathbb{Z}}$ is $NID(0,1)$ and $\{\eta_t\}_{t\in\mathbb{Z}}$ is $NID(0, \sigma_\eta^2)$.

Recall that the indirect inference estimator $\hat{\theta}_{HT}$ is defined as

$$\hat{\theta}_{HT} = \mathrm{argmin}_{\theta\in\Theta} d(\hat{B}_T, \tilde{B}_H(\theta)),$$

where $\hat{B}_T$ is the auxiliary statistic obtained from the observed sample of data and $\tilde{B}_H(\theta)$ is the auxiliary statistic obtained from the simulated sample of data.

Comment on the following statement: ``*The model has 3 parameters to be estimated. Therefore, we need to use 3 auxiliary statistics to estimate the model by indirect inference. For instance, we can use the following 3 statistics: sample mean, sample variance, and first order autocorrelation of* y_t^2". Do you agree with the statement? Justify your answer.

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