vrije Universiteit amsterdam

Preparation Exam
Financial Econometrics

Exam:	Financial Econometrics (3.5)
Code:	-
Coordinator:	dr. P. Gorgi
Co-Reader:	-
Date:	-
Time:	-
Duration:	2 hours and 45 minutes
Calculator:	Allowed
Graphical calculator:	Not allowed
Number of questions:	4
Type of questions:	Open
Answer in:	English
Credit score:	100 credits counts for a 10
Grades:	Made public within 10 working days
Inspection:	-
Number of pages:	-

- Read the entire exam carefully before you start answering the questions.
- Be clear and concise in your statements, but justify every step in your derivations.
- The questions should be handed back at the end of the exam. Do not take it home.

Good luck!

Question 1 [35 points] Observation-Driven Models: Stochastic Properties

Consider the following ARCH(2) model:

$$y_t = \sigma_t \varepsilon_t \quad , \quad \{\varepsilon_t\}_{t\in\mathbb{Z}} \sim \text{NID}(0,1)$$

$$\text{where} \quad \sigma_t^2 = \omega + \alpha_1 y_{t-1}^2 + \alpha_2 y_{t-2}^2 \quad \text{for} \quad t \in \mathbb{Z},$$

where $\omega > 0$, $\alpha_1 \geq 0$, $\alpha_2 \geq 0$ and $\alpha_1 + \alpha_2 < 1$.

(a) Show that y_t has unconditional mean zero; i.e. show that $\mathbb{E}(y_t) = 0$.

(b) Derive the unconditional variance of y_t; i.e. derive an expression for $\mathbb{V}\text{ar}(y_t)$ in terms of the parameters ω, α_1 and α_2.

(c) Suppose that the following *for loop* is used in MATLAB to simulate data from the ARCH(2) model:

```
sig(1)  =  omega/(1-alpha1-alpha2);

for t=2:T

y(t) = sqrt(sig(t)) * epsilon(t);
sig(t+1) = omega + alpha1*y(t)^2 + alpha2*y(t-1)^2;

end
```

Consider the following statement: "*the for loop should start at* `t=1` *because we have specified the initial value* `sig(1)`". Is the statement true or false? Justify your answer.

Consider the following updating equations for the conditional variances and conditional covariance between a bivariate vector $\boldsymbol{y}_t = (y_{1,t}, y_{2,t})^\top$ of stock returns:

$$\begin{bmatrix} \sigma_{1,t}^2 & \sigma_{21,t} \\ \sigma_{21,t} & \sigma_{2,t}^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 0.5 \\ 0.5 & 2 \end{bmatrix} \odot \begin{bmatrix} y_{1,t-1}^2 & y_{1,t-1}y_{2,t-1} \\ y_{1,t-1}y_{2,t-1} & y_{2,t-1}^2 \end{bmatrix}$$

Suppose that the last observed returns for stocks 1 and 2 were given by $y_{1t-1} = 1$ and $y_{2t-1} = 0$. Additionally, consider three different portfolios which assign different weights k_1 and k_2 to stocks 1 and 2 respectively:

<u>Portfolio A</u>: consists only of stock 1 ($k_1 = 1$, $k_2 = 0$)

<u>Portfolio B</u>: consists only of stock 2 ($k_1 = 0$, $k_2 = 1$)

<u>Portfolio C</u>: gives the same weight to each stock ($k_1 = 0.5$, $k_2 = 0.5$)

(d) Which portfolio has lower risk? i.e. which portfolio has lower conditional variance?

(e) Suppose further that $\mu_{1,t} = \mathbb{E}(y_{1,t}|Y^{t-1}) = 1$ and $\mu_{2,t} = \mathbb{E}(y_{2,t}|Y^{t-1}) = 2$. Calculate the Sharpe ratio of each portfolio.

(f) Consider the following statement: *"Portfolio A is the best because it has the lowest Sharpe ratio"*. Is the statement true or false? Justify your answer.

Question 2 [25 points] Observation-Driven Models: Parameter Estimation

Consider the following GARCH(1,1) model:

$$y_t = \sigma_t \varepsilon_t \quad , \quad \{\varepsilon_t\}_{t\in\mathbb{Z}} \sim \text{NID}(0,1)$$

$$\text{where} \quad \sigma_t^2 = \omega + \alpha_1 y_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \quad \text{for} \quad t \in \mathbb{Z}.$$

(a) Consider the following MATLAB code that sets the initial value of the parameter vector $\theta = (\omega, \alpha_1, \beta_1)$ which is used with the `FMINCON` optimization package for obtaining estimates of parameters of the GARCH(1,1) model:

```
theta_ini = [0, 0, 0]
```

Consider the following statement: *"Setting the initial value of the parameter* $\theta = (0,0,0)$ *is problematic"*. Is the statement true or false? Justify your answer.

(b) You have estimated several competing GARCH(p,q) model specifications and obtained the following results for the log likelihood, the Akaike's Information Criterion (AIC) and the Bayesian Information Criterion (BIC):

Model	Log Likelihood	AIC	BIC
GARCH(1,1)	-4781.3	9558.6	9573.4
GARCH(1,2)	-4787.5	9583.0	9589.4
GARCH(2,1)	-4699.1	9406.2	9412.6
GARCH(2,2)	-4698.7	9407.4	9415.4

Are the following statements true or false? Please justify your answer.

(i) *"The GARCH(2,2) is the best model because it has the largest log likelihood value."*

(ii) *"The GARCH(1,2) is the best model because it has the largest AIC and BIC values."*

(iii) *"If the GARCH(2,1) model is better than the GARCH(1,1) model in terms of both AIC and BIC, then this means that the GARCH(2,1) model is well specified."*

Question 3 [20 points] Parameter-Driven Models: Stochastic Properties

(a) Explain and discuss the main differences in the estimation of parameters between *Generalized Autoregressive Conditional Heteroeskedasticity* (GARCH) models and *Stochastic Volatility* (SV) models?

(b) Let $\{y_t\}_{t\in\mathbb{Z}}$ be generated by the following SV-MA(2) model

$$y_t = \sigma_t \epsilon_t, \quad \sigma_t^2 = \exp(f_t),$$
$$f_t = \eta_t + \phi\eta_{t-2},$$

where $\{\epsilon_t\}_{t\in\mathbb{Z}}$ is $NID(0,1)$ and $\{\eta_t\}_{t\in\mathbb{Z}}$ is $NID(0,\sigma_\eta^2)$ with $\sigma_\eta^2 > 0$.

Recall that if z_t is a *normal* random variable $z_t \sim N(\mu,\sigma^2)$, then $\exp(z_t)$ has a *log-normal* distribution, which is denoted $\exp(z_t) \sim \text{log-}N(\mu,\sigma^2)$, and furthermore, has mean given by

$$\mathbb{E}(z_t) = \exp(\mu + \sigma^2/2).$$

(i) Show that the first-order autocorrelation function of y_t^2 is equal to zero, i.e. show that $\mathbb{C}\text{orr}(y_t^2, y_{t-1}^2) = 0$.

(ii) Show that the skewness of y_t is zero, i.e. show that $\mathbb{E}(y_t^3) = 0$.

(iii) Is the unconditional distribution of y_t Normal if $\phi = 0$? Justify your answer.

Question 4 [20 points] Parameter-Driven Models: Parameter Estimation

(a) Consider the indirect inference estimator $\hat{\theta}_{HT}$ given by

$$\hat{\theta}_{HT} = \arg\min_{\theta \in \Theta} d(\hat{B}_T, \tilde{B}_H(\theta)),$$

where $\hat{B}_T$ is the auxiliary statistic obtained from the observed sample of data (which is of length T) and $\tilde{B}_H(\theta)$ is the auxiliary statistic obtained from the simulated sample of data (which is of length H).

Consider the following statement: *"the accuracy of the indirect inference estimator $\hat{\theta}_{HT}$ increases as the length H of the simulations increases".* Is the statement true or false? Justify your answer.

(b) Consider the following SV-MA(1) model

$$y_t = \sigma_t \varepsilon_t, \quad \sigma_t^2 = \exp(f_t),$$
$$f_t = \eta_t + \phi \eta_{t-1},$$

where $\{\varepsilon_t\}_{t \in \mathbb{Z}}$ is $NID(0,1)$ and $\{\eta_t\}_{t \in \mathbb{Z}}$ is $NID(0,1)$. The parameter vector is given by $\theta = \phi$ and the parameter set is $\Theta = (-1,1)$.

We want to estimate the "true" parameter $\theta_0 = \phi_0 \in (-1,1)$ by indirect inference. Consider the following auxiliary statistics

$$\hat{B}_T = \frac{1}{T} \sum_{t=2}^{T} y_t y_{t-1}, \quad \text{and} \quad \tilde{B}_H(\theta) = \frac{1}{H} \sum_{t=2}^{H} \tilde{y}_t(\theta) \tilde{y}_{t-1}(\theta)$$

Show whether or not the indirect inference estimator $\hat{\theta}_{HT}$ based on the above auxiliary statistics is consistent.