1. The current price of gold is 1700 USD per ounce, and the storage costs are 1% per year of the total value of gold stored. Assume annual interest rates are 1.5%.

   a) What is the futures price of gold (per ounce) for delivery in six months?

   b) Three months later, the gold price went up by 10% and the interest rates remained the same. What is the current futures price for delivery in 3 months and what is the value of the forward contract in a)?

2. Today's Eurodollar exchange rate is 1.31 (1 Euro = 1.31 USD), US risk free rate is 2% and ECB risk free rate is 1.5%.

   a) What is the 6-months forward exchange rate and the price of a six-month forward contract for 1M USD?

   b) If, in one month, the Euro falls 5% with respect to USD and the interest rates remain unchanged, what is then the three-month forward exchange rate and the value of the forward contract in a)?

3. The price of a one-month European call option on a non-dividend paying stock is 3$. The stock price is 47$, the strike is 50$, the risk-free rate is 3% per year. The price of the put option with the same maturity and the strike is 6.3$. Is there an arbitrage opportunity? How would you realize it?

4. A stock price is currently \$ 30 and at the end of the three-month period it can either go up by 10 % or down by 8 %.

   What is the price of the ATM European call option on this stock, maturing in three months? What are the risk-neutral probabilities and the option's delta at time zero? Assume the risk free continuously compound interest rate of 4% per annum.

5. A stock price is currently \$ 50 and at the end of each three-month period it can either go up by \$ 5 or down by \$ 4.

   a) We are interested in the ATM European put option maturing in six months time, i.e., after two periods of three months. With the help of two-step binomial tree, calculate the risk-neutral probabilities, the price of such a call option, its value in each node of the tree and all the option's deltas. Again, assume zero interest rate.

   b) Calculate the price of a 6-months ATM *American* put option. Should it be exercised early?

   c) With the same two-period binomial tree as in b), compute the price of a digital call, i.e., the option that pays \$ 1 if the final stock price after six months is above the initial stock price (of \$ 50).

6. Recall that the Black-Scholes formula for European call option with strike K and time to maturity T, on a stock with price S_0 that pays dividend yield at the rate q per annum is

$$c = S_0 e^{-qT} N(d_1) - K e^{-rT} N(d_2),$$

   where

$$d_1 = \frac{\ln(S_0/K) + (r - q + \sigma^2/2)T}{\sigma\sqrt{T}}, \quad d_2 = d_1 - \sigma\sqrt{T}.$$

   Consider a European call option maturing in six months, on a non-dividend paying stock with the current price of \$ 47, strike price of \$ 50, interest rate 3% p/a and volatility of 30% p/a.

   a) Calculate the price of such an option.

b) What is the expression for the delta of the option in a) and what is the value of delta?