Extra Exam Empirical Methods

VU University Amsterdam, Faculty of Exact Sciences

12.00 – 14.45, July 2, 2015

- *Also hand in this exam and your scrap paper.*
- Always *motivate* your answers.
- Write your answers in English.
- Only the use of a simple, non-graphical calculator is allowed.
- Programmable/graphical calculators, laptops, mobile phones, smart watches, books, own formula sheets, etc. are not allowed.
- On the last four pages of the exam, some formulas and tables that you may want to use can be found.
- The total number of points you can receive is 90: Grade $= 1 + \frac{\text{points}}{10}$.
- The division of points per question and subparts is as follows:

Question	1	2	3	4	5	6
Part (a)	3	3	3	1	1	3
Part (b)	3	3	5	8	4	4
Part (c)	3	4	6	2	5	3
Part (d)	3	4	-	8	3	3
Part (e)	-	-	-	3	2	-
Total	12	14	14	23	15	13

- If you are asked to perform a test, do not only give the conclusion of your test, but report:
  1. the hypotheses in terms of the population parameter of interest;
  2. the significance level;
  3. the test statistic and its distribution under the null hypothesis;
  4. the observed value of the test statistic;
  5. the P-value or the critical value(s);
  6. whether or not the null hypothesis is rejected and why;
  7. finally, phrase your conclusion in terms of the context of the problem.

1. For the following situations identify which of the following applies: simple random sample, systematic sample, convenience sample, stratified sample, or cluster sample. In each case, state whether you think the procedure is likely to yield a representative sample or a biased sample, and briefly explain why.

   (a) People magazine chooses its "best dressed celebrities" by compiling responses from readers who mailed the magazine their answers to the questions in a survey that was printed in the magazine.

   (b) A marketing expert for MTV is planning a survey in which 500 people will be randomly selected from each age group of 10-19, 20-29, and so on.

   Determine whether the data described in parts (c) and (d) are qualitative or quantitative and give their level of measurement. Indicate also which type of visualization is most suited for these data and why.

   (c) A question in a survey has five possible answers, 1, 2, 3, 4, and 5, which stand for very unhappy, unhappy, neutral, happy, and very happy, respectively. The data consist of the answers to this question of 150 people.

   (d) With carbon dating, the ages (in years) of 78 specimens of wood were determined.

2. From a study conducted three years ago it was concluded that 80% of the people had "good" financial credit ratings, while the remaining 20% had "bad" financial credit ratings. Current records show that 30% of those with bad credit ratings have since improved their ratings to good, while 15% of those with good credit ratings have since changed to having bad credit ratings.

   In the items below, do not only give your answer, but also show how you obtained it and name the rule(s) or property(ies) of probabilities that you have used for its computation.

   (a) What is the probability that two randomly chosen people from the population had good credit ratings three years ago?

   (b) What is the probability that at least one of three randomly chosen people had bad credit ratings three years ago?

   (c) What is the probability that a randomly selected person now has good ratings?

   (d) What is the probability that a person who has good ratings now, had bad ratings three years ago?

3. Assume that systolic blood pressure is a normally distributed random variable with mean $\mu_{\text{sys}} = 121$ and standard deviation $\sigma_{\text{sys}} = 15$.

   (a) What is the probability that the systolic blood pressure of a randomly selected person is larger than 140?

   (b) What is the probability that the mean of the systolic blood pressures of 25 randomly selected people is between 118 and 126?

   Now assume that the diastolic blood pressure in a population is a normally distributed random variable with unknown mean μ_{dia} and known standard deviation $\sigma_{\text{dia}} = 15$. Suppose that the diastolic blood pressure in a sample of 16 randomly selected people from that population is measured and that the sample mean is

$\overline{x} = 87.0$. We want to investigate the conjecture that the unknown population mean μ_{dia} is larger than 81.

(c) Use a suitable statistical test to investigate this conjecture. Take significance level $\alpha = 0.05$.

4. Listed below are two samples of body temperatures (in °C):

sample 1:	36.1	35.7	36.4	35.8	36.6	37.3;
sample 2:	36.7	37.0	37.1	36.7	37.0	36.4.

The sample means and sample standard deviations for these data are:

$$\begin{aligned} \text{for sample 1} &: \overline{x}_1 = 36.3, s_1 = 0.59; \\ \text{for sample 2} &: \overline{x}_2 = 36.8, s_2 = 0.26; \\ \text{for the pairwise differences} &: \overline{d} = -0.5, s_d = 0.75. \end{aligned}$$

Some other numbers that you may or may not use:

$$s_p\sqrt{1/n_1 + 1/n_2} = 0.26, \qquad \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} = 0.26.$$

First assume that the temperatures were measured on 6 subjects, at 8.00h and at 12.00h in the morning, resulting in sample 1 and sample 2, respectively.

(a) Give an estimate of the difference in mean temperature at 8.00h and 12.00h.

(b) Use a suitable t-test (motivate your choice!) to investigate the claim that on average the temperatures at 8.00h are lower than those at 12.00h. Take significance level $\alpha = 0.05$.

Now assume instead that all temperatures were measured at 8.00h, with sample 1 containing the temperatures of 6 men and sample 2 containing the temperatures of 6 women.

(c) Give an estimate of the difference in mean temperature at 8.00h of men and women.

(d) Use a suitable t-test (motivate your choice!) to investigate the claim that the mean temperatures at 8.00h of men and women are different. Take significance level $\alpha = 0.05$.

(e) Is the use of a t-test in parts (b) and (d) justifiable? Motivate your answer.

5. In a poll among 100 randomly selected VU-students about whether or not they agree with the statement that students who have more than 12,000 euros extra earnings should not obtain a scholarship, 8 students agreed with the statement and 92 students disagreed. Let p be the fraction of VU-students who agree with the statement.

(a) Determine the usual point estimate $\hat{p}$ for p.

(b) What is the interpretation of a 95% confidence interval for the unknown population proportion p?

(c) Give the 95% confidence interval based on the results of the poll among the 100 students.

(d) In general, to estimate p with a 95% degree confidence and a margin of error of 0.05, what should be the minimum sample size?

(e) Compare your answer and the margin of error of part (d) with the size of the poll and the margin of error that you used to compute the interval in part (c). Can you explain the differences/similarities?

6. In each province a number of randomly selected people were asked whether or not they think that the appearance of Zwarte Piet should change. The results for Limburg, Groningen, and Noord-Holland are given in the following table.

	change	no change	total
Limburg	3	147	150
Groningen	6	194	200
Noord-Holland	22	228	250
total	31	569	600

(a) Use the table to give, for each of the three provinces separately and under the assumption that there is no relationship between the variables 'province' and 'change', the expected number of people in the sample from that province who think that the appearance of Zwarte Piet should change.

(b) Suppose that we wish to investigate with a chi-square test whether or not there is a relationship between the variables 'province' and 'change'. Formulate suitable hypotheses, specify the test statistic (also tell what the symbols that you use in the formula that you give for the test statistic, stand for), and its distribution under the null hypothesis. (You do not need to compute the observed value of the test statistic.)

(c) The observed value of the test statistic for these data is 11.72. What would be the conclusion of the test that you described in part (b) for significance level $\alpha = 1\%$? Motivate your answer.

(d) The test that you described in part (b) should only be used when certain requirements are met. What are these requirements and are they satisfied in this case?

Formulas and Tables for Exam Empirical Methods

Probability

We use the following notation:
Ω sample space, P probability measure.
$B, A_1, A_2, \dots, A_m$ events,
$A_1, A_2, \dots, A_m$ a partition of Ω with $P(A_i) > 0$ for all $i \in \{1, 2, \dots, m\}$.

Law of Total Probability:

$$P(B) = \sum_{i=1}^{m} P(B \cap A_i) = \sum_{i=1}^{m} P(B|A_i)P(A_i).$$

Bayes' Theorem:

$$P(A_r|B) = \frac{P(A_r \cap B)}{\sum_{i=1}^{m} P(B|A_i)P(A_i)} = \frac{P(B|A_r)P(A_r)}{\sum_{i=1}^{m} P(B|A_i)P(A_i)}.$$

Two *independent* samples

(The statements below hold if certain requirements are met.)

For two *independent* samples,
(i) if σ_1 and σ_2 are unknown and $\sigma_1 \neq \sigma_2$, the test statistic

$$T_2 = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{s_1^2/n_1 + s_2^2/n_2}}$$

has a t-distribution with approximately $\tilde{n}$ degrees of freedom under the null hypothesis. We use the conservative estimate $\tilde{n} = \min\{n_1 - 1, n_2 - 1\}$.
(ii) if σ_1 and σ_2 are unknown and $\sigma_1 = \sigma_2$, then the test statistic

$$T_2^{\text{eq}} = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{s_p^2/n_1 + s_p^2/n_2}}$$

has a t-distribution with $n_1 + n_2 - 2$ degrees of freedom under the null hypothesis. Here s_p is the square root of the pooled sample variance s_p^2 given by

$$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}.$$

(iii) if σ_1 and σ_2 are known, then the test statistic

$$Z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\sigma_1^2/n_1 + \sigma_2^2/n_2}}$$

has a standard normal distribution under the null hypothesis.
(iv) if $p_1 = p_2$, the test statistic

$$Z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\bar{p}(1-\bar{p})/n_1 + \bar{p}(1-\bar{p})/n_2}}$$

approximately has a standard normal distribution. Here $\bar{p} = (x_1+x_2)/(n_1+n_2)$ is the pooled sample proportion.
(v) the margin of error for a $1-\alpha$ confidence interval for $p_1 - p_2$ is given by

$$E = z_{\alpha/2}\sqrt{\hat{p}_1(1-\hat{p}_1)/n_1 + \hat{p}_2(1-\hat{p}_2)/n_2}.$$

Correlation

Under certain conditions the test statistic

$$T_{cor} = \frac{r-\rho}{\sqrt{(1-r^2)/(n-2)}}$$

has a t-distribution with $n-2$ degrees of freedom. Here ρ is the population linear correlation coefficient and r is the sample linear correlation coefficient given by

$$r = \frac{1}{n-1}\sum_{i=1}^{n}\Big[\frac{(x_i-\bar{x})(y_i-\bar{y})}{s_x s_y}\Big].$$

Linear regression

Let β_0 be the unknown intercept and β_1 the unknown slope of a linear regression model with one explanatory variable, and let b_0 and b_1 be the corresponding estimators, i.e. the intercept and slope of the regression line (the 'best' line). Then b_0 and b_1 are given by

$$b_1 = r\frac{s_y}{s_x}$$

and

$$b_0 = \bar{y} - b_1\bar{x}.$$

If certain requirements are met, then the test statistic

$$T_1 = \frac{b_1 - \beta_1}{s_{b_1}}$$

has a t-distribution with $n-2$ degrees of freedom. Here s_{b_1} is the standard error (i.e. estimated standard deviation) of the estimator b_1.

NEGATIVE *z* Scores

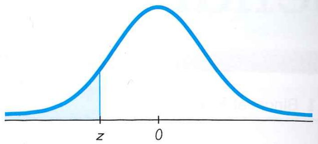


Table 2 Standard Normal (*z*) Distribution: Cumulative Area from the LEFT

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.50 and lower	.0001									
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051 *	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505 *	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

NOTE: For values of *z* below 3.49, use 0.0001 for the area.
*Use these common values that result from interpolation:

z Score	Area
−1.645	0.0500
−2.575	0.0050

(*continued*)

POSITIVE *z* Scores

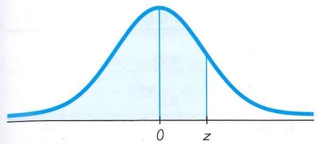


Table 2 *(continued)* Cumulative Area from the LEFT

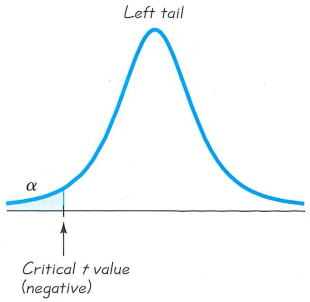
z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495 *	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949 *	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998
3.50 and up	.9999									

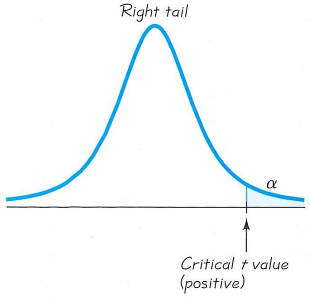
NOTE: For values of *z* above 3.49, use 0.9999 for the area.
*Use these common values that result from interpolation:

z Score	Area
1.645	0.9500
2.575	0.9950

Common Critical Values

Confidence Level	Critical Value
0.90	1.645
0.95	1.96
0.99	2.575





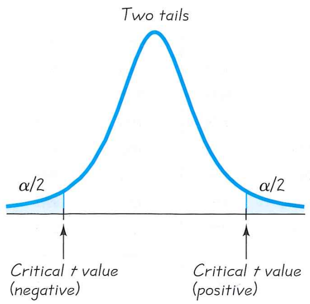


Table 3 *t* Distribution: Critical *t* Values

	Area in One Tail				
	0.005	0.01	0.025	0.05	0.10
	Area in Two Tails				
Degrees of Freedom	0.01	0.02	0.05	0.10	0.20
1	**63.657**	31.821	**12.706**	6.314	3.078
2	**9.925**	6.965	**4.303**	2.920	1.886
3	**5.841**	4.541	**3.182**	2.353	1.638
4	**4.604**	3.747	**2.776**	2.132	1.533
5	**4.032**	3.365	**2.571**	2.015	1.476
6	**3.707**	3.143	**2.447**	1.943	1.440
7	**3.499**	2.998	**2.365**	1.895	1.415
8	**3.355**	2.896	**2.306**	1.860	1.397
9	**3.250**	2.821	**2.262**	1.833	1.383
10	**3.169**	2.764	**2.228**	1.812	1.372
11	**3.106**	2.718	**2.201**	1.796	1.363
12	**3.055**	2.681	**2.179**	1.782	1.356
13	**3.012**	2.650	**2.160**	1.771	1.350
14	**2.977**	2.624	**2.145**	1.761	1.345
15	**2.947**	2.602	**2.131**	1.753	1.341
16	**2.921**	2.583	**2.120**	1.746	1.337
17	**2.898**	2.567	**2.110**	1.740	1.333
18	**2.878**	2.552	**2.101**	1.734	1.330
19	**2.861**	2.539	**2.093**	1.729	1.328
20	**2.845**	2.528	**2.086**	1.725	1.325
21	**2.831**	2.518	**2.080**	1.721	1.323
22	**2.819**	2.508	**2.074**	1.717	1.321
23	**2.807**	2.500	**2.069**	1.714	1.319
24	**2.797**	2.492	**2.064**	1.711	1.318
25	**2.787**	2.485	**2.060**	1.708	1.316
26	**2.779**	2.479	**2.056**	1.706	1.315
27	**2.771**	2.473	**2.052**	1.703	1.314
28	**2.763**	2.467	**2.048**	1.701	1.313
29	**2.756**	2.462	**2.045**	1.699	1.311
30	**2.750**	2.457	**2.042**	1.697	1.310
31	**2.744**	2.453	**2.040**	1.696	1.309
32	**2.738**	2.449	**2.037**	1.694	1.309
33	**2.733**	2.445	**2.035**	1.692	1.308
34	**2.728**	2.441	**2.032**	1.691	1.307
35	**2.724**	2.438	**2.030**	1.690	1.306
36	**2.719**	2.434	**2.028**	1.688	1.306
37	**2.715**	2.431	**2.026**	1.687	1.305
38	**2.712**	2.429	**2.024**	1.686	1.304
39	**2.708**	2.426	**2.023**	1.685	1.304
40	**2.704**	2.423	**2.021**	1.684	1.303
45	**2.690**	2.412	**2.014**	1.679	1.301
50	**2.678**	2.403	**2.009**	1.676	1.299
60	**2.660**	2.390	**2.000**	1.671	1.296
70	**2.648**	2.381	**1.994**	1.667	1.294
80	**2.639**	2.374	**1.990**	1.664	1.292
90	**2.632**	2.368	**1.987**	1.662	1.291
100	**2.626**	2.364	**1.984**	1.660	1.290
200	**2.601**	2.345	**1.972**	1.653	1.286
300	**2.592**	2.339	**1.968**	1.650	1.284
400	**2.588**	2.336	**1.966**	1.649	1.284
500	**2.586**	2.334	**1.965**	1.648	1.283
1000	**2.581**	2.330	**1.962**	1.646	1.282
2000	**2.578**	2.328	**1.961**	1.646	1.282
Large	**2.576**	2.326	**1.960**	1.645	1.282

Table 4 Chi-Square (χ^2) Distribution

	Area to the Right of the Critical Value									
Degrees of Freedom	0.995	0.99	0.975	0.95	0.90	0.10	0.05	0.025	0.01	0.005
1	—	—	0.001	0.004	0.016	2.706	3.841	5.024	6.635	7.879
2	0.010	0.020	0.051	0.103	0.211	4.605	5.991	7.378	9.210	10.597
3	0.072	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.345	12.838
4	0.207	0.297	0.484	0.711	1.064	7.779	9.488	11.143	13.277	14.860
5	0.412	0.554	0.831	1.145	1.610	9.236	11.071	12.833	15.086	16.750
6	0.676	0.872	1.237	1.635	2.204	10.645	12.592	14.449	16.812	18.548
7	0.989	1.239	1.690	2.167	2.833	12.017	14.067	16.013	18.475	20.278
8	1.344	1.646	2.180	2.733	3.490	13.362	15.507	17.535	20.090	21.955
9	1.735	2.088	2.700	3.325	4.168	14.684	16.919	19.023	21.666	23.589
10	2.156	2.558	3.247	3.940	4.865	15.987	18.307	20.483	23.209	25.188
11	2.603	3.053	3.816	4.575	5.578	17.275	19.675	21.920	24.725	26.757
12	3.074	3.571	4.404	5.226	6.304	18.549	21.026	23.337	26.217	28.299
13	3.565	4.107	5.009	5.892	7.042	19.812	22.362	24.736	27.688	29.819
14	4.075	4.660	5.629	6.571	7.790	21.064	23.685	26.119	29.141	31.319
15	4.601	5.229	6.262	7.261	8.547	22.307	24.996	27.488	30.578	32.801
16	5.142	5.812	6.908	7.962	9.312	23.542	26.296	28.845	32.000	34.267
17	5.697	6.408	7.564	8.672	10.085	24.769	27.587	30.191	33.409	35.718
18	6.265	7.015	8.231	9.390	10.865	25.989	28.869	31.526	34.805	37.156
19	6.844	7.633	8.907	10.117	11.651	27.204	30.144	32.852	36.191	38.582
20	7.434	8.260	9.591	10.851	12.443	28.412	31.410	34.170	37.566	39.997
21	8.034	8.897	10.283	11.591	13.240	29.615	32.671	35.479	38.932	41.401
22	8.643	9.542	10.982	12.338	14.042	30.813	33.924	36.781	40.289	42.796
23	9.260	10.196	11.689	13.091	14.848	32.007	35.172	38.076	41.638	44.181
24	9.886	10.856	12.401	13.848	15.659	33.196	36.415	39.364	42.980	45.559
25	10.520	11.524	13.120	14.611	16.473	34.382	37.652	40.646	44.314	46.928
26	11.160	12.198	13.844	15.379	17.292	35.563	38.885	41.923	45.642	48.290
27	11.808	12.879	14.573	16.151	18.114	36.741	40.113	43.194	46.963	49.645
28	12.461	13.565	15.308	16.928	18.939	37.916	41.337	44.461	48.278	50.993
29	13.121	14.257	16.047	17.708	19.768	39.087	42.557	45.722	49.588	52.336
30	13.787	14.954	16.791	18.493	20.599	40.256	43.773	46.979	50.892	53.672
40	20.707	22.164	24.433	26.509	29.051	51.805	55.758	59.342	63.691	66.766
50	27.991	29.707	32.357	34.764	37.689	63.167	67.505	71.420	76.154	79.490
60	35.534	37.485	40.482	43.188	46.459	74.397	79.082	83.298	88.379	91.952
70	43.275	45.442	48.758	51.739	55.329	85.527	90.531	95.023	100.425	104.215
80	51.172	53.540	57.153	60.391	64.278	96.578	101.879	106.629	112.329	116.321
90	59.196	61.754	65.647	69.126	73.291	107.565	113.145	118.136	124.116	128.299
100	67.328	70.065	74.222	77.929	82.358	118.498	124.342	129.561	135.807	140.169

Source: Donald B. Owen, *Handbook of Statistical Tables.*

Degrees of Freedom

$n - 1$	**Confidence interval or hypothesis test** for a standard deviation σ or variance σ^2
$k - 1$	**Goodness-of-fit** with k categories
$(r - 1)(c - 1)$	**Contingency table** with r rows and c columns
$k - 1$	**Kruskal-Wallis test** with k samples