Econometrics III: Exam 2020

Problem 1 (25/60 points)

Consider the following VAR(2) model for $\mathbf{y}_t = (y_{1t}, y_{2t})'$:

$$\mathbf{y}_t = \begin{pmatrix} 0.1 \\ 0 \end{pmatrix} + \begin{pmatrix} \alpha_1 & \beta_1 \\ \gamma_1 & \delta_1 \end{pmatrix} \mathbf{y}_{t-1} + \begin{pmatrix} \alpha_2 & \beta_2 \\ \gamma_2 & \delta_2 \end{pmatrix} \mathbf{y}_{t-2} + \mathbf{u}_t, \quad t = 0, \pm 1, \pm 2, ... \tag{1}$$

The innovations $\mathbf{u}_t = (u_{1t}, u_{2t})'$ are assumed to be generated by an uncorrelated Gaussian process with mean zero and fixed, nonsingular covariance matrix $\Sigma_{\mathbf{u}}$.

(a) (**5 points**) Write down the companion form of model (1) and give the general condition for stability. Assume in the following that the process is stable.

(b) (**4 points**) Is the VAR process (1) weakly stationary? Is it strictly stationary? Explain briefly.

(c) (**9 points**)

  (i) Write down model (1) imposing the parameter restriction(s) implying that y_{1t} does not Granger-cause y_{2t}.

  (ii) Under the restriction in part (i), compute the unconditional mean of $\mathbf{y}_t$. Simplify as much as possible. Hint: Your final answer should be a (2×1) vector.

(d) (**4 points**) Stable VAR processes are often expressed in infinite vector moving average (MA) form. Briefly describe two situations in which the MA representation is useful in multiple time series analysis.

(e) (**3 points**) Briefly describe the intuition underlying the use of information criteria such as AIC, BIC/SC and HQ for model selection in multiple time series analysis.

Problem 2 (20/60 points)

Consider a zero-mean vector error correction model of dimension one, i.e. a VECM(1), for a 2-dimensional vector of variables $\mathbf{y}_t = (y_{1t}, y_{2t})'$:

$$\Delta \mathbf{y}_t = \Pi \mathbf{y}_{t-1} + \Gamma \Delta \mathbf{y}_{t-1} + \mathbf{u}_t \tag{2}$$

where Π and Γ are (2×2)-coefficient matrices. The innovations $\mathbf{u}_t = (u_{1t}, u_{2t})'$ are assumed to be generated by an uncorrelated Gaussian process with mean zero and identity covariance matrix

$$\Sigma_{\mathbf{u}} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

(a) (**2 points**) Suppose both variables y_{1t} and y_{2t} are integrated of order one. Under which condition on the coefficient matrices in model (2) are the variables cointegrated?

(b) (**3 points**) Briefly explain why it is useful to test for cointegration when analyzing a multiple time series with components that are integrated of order one.

(c) (**6 points**) Suppose T observations plus a suitable number of pre-sample observations are available. Define

$$\begin{aligned}
\Delta \mathbf{Y} &= (\Delta \mathbf{y}_1 : \ldots : \Delta \mathbf{y}_T), \\
\mathbf{Y}_{-1} &= (\mathbf{y}_0 : \ldots : \mathbf{y}_{T-1}), \\
\Delta \mathbf{X} &= (\Delta \mathbf{y}_0 : \ldots : \Delta \mathbf{y}_{T-1}), \\
\mathbf{U} &= (\mathbf{u}_1 : \ldots : \mathbf{u}_T)
\end{aligned}$$

(i) Write model (2) jointly for all observations in matrix notation.

(ii) State the distribution as well as the expectation and covariance matrix of the vectorized matrix of disturbances $\text{vec}(\mathbf{U})$. Include the matrix dimensions.

(d) (**9 points**) The likelihood function to estimate the parameters in model (2) is given by

$$\ell = -\frac{KT}{2}\ln(2\pi) - \frac{1}{2}\text{tr}\left[(\Delta \mathbf{Y} - \Pi \mathbf{Y}_{-1} - \Gamma \Delta \mathbf{X})'(\Delta \mathbf{Y} - \Pi' \mathbf{Y}_{-1} - \Gamma \Delta \mathbf{X})\right].$$

where tr denotes the trace operator. Simplify the estimation problem by applying a suitable transformation that eliminates the coefficient matrix Γ from the likelihood function. Write down the concentrated likelihood. If you introduce new model components, make sure they are defined properly.

Problem 3 (15/60 points)

Consider the one-way error component panel data model for a sample on N individuals at T time points in matrix notation:

$$y = \alpha \mathbf{1}_{NT} + X\beta + \epsilon, \quad \epsilon = G\mu + e, \quad e \sim N(0, \sigma_e^2 \mathrm{I}_{NT}) \tag{3}$$

- $\beta = (\beta_1, ..., \beta_K)'$ is a vector of unknown fixed coefficients,
- α is an unknown fixed intercept,
- μ is a $(N \times 1)$-vector of individual-specific effects,
- X is a $(NT \times K)$-matrix of non-random regressors,
- y and e are $(NT \times 1)$-vectors of dependent variable observations and disturbances, respectively,
- $\mathbf{1}_{NT}$ denotes a $(NT \times 1)$-vector of ones
- $G = \mathrm{I}_N \otimes \mathbf{1}_T$ is a matrix of individual dummies and I_N denotes the identity matrix of dimension N.

(a) (**4 points**) Compute the rank of the projection matrix $M_G = \mathrm{I}_{NT} - G(G'G)^{-1}G'$.

(b) (**5 points**) Define

$$P_G = G(G'G)^{-1}G' \quad \text{and} \quad P_0 = \mathbf{1}_{NT}(\mathbf{1}_{NT}'\mathbf{1}_{NT})^{-1}\mathbf{1}_{NT}'.$$

Explain how the elements in the vector of dependent variable observations y are transformed when y is pre-multiplied by $(P_G - P_0)$.

(c) (**6 points**) Briefly explain the one potential advantage and one potential disadvantage when using the generalized least squares (GLS) estimator to estimate β in model (3).