Ectr. III Example Exam 2

Problem 1 (35/90 points)

Suppose a model for an industrial sector's profit growth (y_1) and investment growth (y_2) is given by the bivariate vector autoregressive (VAR) process for $\mathbf{y}_t = (y_{1t}, y_{2t})'$:

$$\mathbf{y}_t = \begin{pmatrix} 0.2 \\ 0.1 \end{pmatrix} + \begin{pmatrix} 0.8 & 0.2 \\ 0 & 0.5 \end{pmatrix} \mathbf{y}_{t-1} + \mathbf{u}_t, \quad t = 0, \pm 1, \pm 2, ... \tag{1}$$

The innovations $\mathbf{u}_t = (u_{1t}, u_{2t})'$ are assumed to be generated by an uncorrelated Gaussian process with mean zero and fixed, nonsingular covariance matrix,

$$\Sigma_{\mathbf{u}} = \begin{pmatrix} 0.3 & 0 \\ 0 & 0.6 \end{pmatrix}.$$

(a) (5 points) Show that the process (1) is stable.

(b) (6 points) Compute the unconditional mean of y_{1t} (profit growth).

(c) (4 points) Does profit growth Granger-cause investment growth? Does investment growth Granger-cause profit growth? Justify briefly.

(d) (7 points) Compute the impulse responses of the VAR given in (1) for horizons $h = 0, 1, 2$. Given a unit shock to profit growth in $t = 0$, what effect on investment growth do you expect as $h \to \infty$?

(e) (3 points) If you wanted to consider orthogonalized impulse responses, would they be different from the ones you computed in (d)? Why or why not?

(f) (10 points) Suppose that at time T, the observed values $\mathbf{y}_T = (y_{T1}, y_{T2})'$ are $(0.4, 0)'$.

- (i) Compute the 1-step and the 2-step-ahead predictions $\mathbf{y}_T(1)$ and $\mathbf{y}_T(2)$.
- (ii) Compute the forecast error variance matrix for the 2-step ahead prediction of $\mathbf{y}_t$.
- (iii) Compute a 95% prediction interval for the 2-step ahead prediction of y_1 (profit growth).

Problem 2 (25/90 points)

Suppose you want to analyze a time series data set on three exchange rate series y_1, y_2 and y_3. Unit root tests have shown that all three time series are $I(1)$.

(a) (10 points) Economic theory suggests that two equilibrium relationships exist between the three variables, implying that $y_1 - 0.3y_2$ is stationary and $y_1 - 2y_3$ is stationary.

(i) Set up a zero-mean vector-error correction model without lagged differences for the vector $\mathbf{y}_t = (y_{1t}, y_{2t}, y_{3t})'$, incorporating the information from economic theory. Give the dimensions of all model components.

(ii) A common normalization for the cointegration matrix is $\beta' = (\mathrm{I}_r : \widetilde{\beta}'_{K-r})$, where r is the number of cointegration relations, I_r is the identity matrix of dimension r, and K is the number of variables in the system. $\widetilde{\beta}_{K-r}$ is a $((K-r) \times r)$ matrix of coefficients. Rewrite the VECM you set up in part (i) in terms of this normalization. *Hint: Consider the ordering of the variables.*

(b) (9 points) Suppose you believe the economic theory regarding β, but you want to estimate the loading matrix α of your VECM in part (a). Suppose T observations plus a suitable number of pre-sample observations of $\mathbf{y}_t$ are available. Define

$$\begin{aligned}
\Delta\mathbf{Y} &= (\Delta\mathbf{y}_1 : \ldots : \Delta\mathbf{y}_T), \\
\mathbf{Y}_{-1} &= (\mathbf{y}_0 : \ldots : \mathbf{y}_{T-1}), \\
\mathbf{U} &= (\mathbf{u}_1 : \ldots : \mathbf{u}_T)
\end{aligned}$$

Write down the model in matrix notation. Show that the least squares estimator for the loading matrix α can be expressed as

$$\widehat{\alpha} = \Delta\mathbf{Y}\mathbf{Y}'_{-1}\beta(\beta'\mathbf{Y}_{-1}\mathbf{Y}'_{-1}\beta)^{-1} \tag{2}$$

(c) (6 points) Suppose the number of cointegration relations for the three variables is unknown and you want to find out about it using a test. State an suitable test sequence by mentioning the test type, null- and alternative hypotheses and the decision rule(s).

Problem 3 (10/90 points)

Consider a dynamic factor model for a $(K \times 1)$-vector of time series Y_t, given by the set of equations

$$\begin{aligned} Y_t &= \Lambda_1 f_t + \Lambda_2 f_{t-1} + \epsilon_t, \qquad \epsilon_t \sim i.i.d.N(0, \Sigma_\epsilon), \quad (3) \\ f_t &= \varphi f_{t-1} + u_t, \qquad u_t \sim i.i.d.N(0, \Sigma_u), \quad (4) \end{aligned}$$

where f_t is a (2×1)-vector of unobserved factors, Λ_1, Λ_2, and φ are matrices of coefficients. Σ_ϵ and Σ_u are diagonal matrices of full rank.

(a) (4 points) What are the dimensions of the matrices Λ_1, Λ_2, φ, Σ_ϵ and Σ_u in (3) and (4)?

(b) (3 points) Re-write equation (3) in the form of a static factor model, i.e.,

$$Y_t = \mathbf{\Lambda} F_t + \epsilon_t.$$

Define all model components appropriately.

(c) (3 points) Re-write equation (4) in the form

$$F_t = \boldsymbol{\psi} F_{t-1} + G u_t, \quad (5)$$

with F_t from (a), where G is a deterministic matrix containing zeros and ones, that makes sure (4) and (5) are equivalent. Define the model components appropriately, including G.

Problem 4 (20/90 points)

Consider the one-way error component panel data model for a sample on N individuals at T time points in matrix notation:

$$y = X\beta + G\mu + e \quad e|X \sim N(0, \sigma_e^2 \mathrm{I}_{NT}) \tag{6}$$

where

- $\beta = (\beta_1, ..., \beta_K)'$ is a vector of unknown fixed coefficients,
- μ is a $(N \times 1)$-vector of unknown fixed individual-specific intercepts,
- X is a $(NT \times K)$-matrix of strictly exogenous random regressors,
- y and e are $(NT \times 1)$-vectors of dependent variable observations and disturbances, respectively,
- $G = \mathrm{I}_N \otimes \mathbb{1}_T$ is a matrix of individual dummies. $\mathbb{1}_T$ denotes a $(T \times 1)$-vector of ones and I_N denotes the identity matrix of dimension N.

(a) (6 points) Define the matrices $P := G(G'G)^{-1}G'$ and $Q := \mathrm{I}_{NT} - P$. Transform model (6) by pre-multiplying both sides of the equation by Q. What are the properties of the transformed data vector Qy and the transformed regressor matrix QX? Do you consider the OLS estimator in this transformed model a good choice to estimate the unknown parameter vector β? Explain briefly.

(b) (10 points) Derive the conditional expectation and the conditional covariance matrix of the within estimator

$$\widehat{\beta}_W = (X'QX)^{-1}X'Qy, \tag{7}$$

where Q is given in part (a), conditional on the regressor matrix X.

(c) (4 points) Briefly explain the purpose of individual-specific fixed effects in a model for panel data.

Appendix

Rules for Kronecker product and vectorization (vec) operator
Assume that the matrices A, B, C, and I (identity matrix) have dimensions such that all products exist.

(K1) $(A \otimes B)' = A' \otimes B'$

(K2) $(A \otimes B)^{-1} = A^{-1} \otimes B^{-1}$

(K3) $(A \otimes B)(C \otimes D) = (AC) \otimes (BD)$

(K4) $\text{vec}(ABC) = (C' \otimes A)\text{vec}(B)$

(K5) $\text{vec}(AB) = (\text{I} \otimes A)\text{vec}(B) = (B' \otimes \text{I})\text{vec}(A)$

(K6) $\text{vec}(B')'\text{vec}(A) = \text{vec}(A')'\text{vec}(B) = \text{tr}(BA) = \text{tr}(AB)$

(K7) $\text{tr}(ABC) = \text{vec}(A')'(\text{I} \otimes B)\text{vec}(C)$