Uitwerking tentamen Dyn & Cemp 21.3.2016

1a

$$\frac{d}{dt}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 & 6 \\ 2 & -2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$$

$\det(A - \lambda I) = 0$ geeft

$$(2-\lambda)(-2-\lambda) - 12 = 0$$

$$\lambda^2 = 16 \iff \lambda = \pm 4 \qquad \tfrac{1}{2}\text{ pt}$$

Eigenvectoren:

$$\lambda = 4: \quad A - 4I = \begin{pmatrix} -2 & 6 \\ 2 & -6 \end{pmatrix} \sim \begin{pmatrix} -1 & 3 \\ 0 & 0 \end{pmatrix}. \quad \underline{v}_1 = \begin{pmatrix} 3 \\ 1 \end{pmatrix} \qquad \tfrac{1}{2}\text{ pt}$$

$$\lambda = -4 \quad A + 4I = \begin{pmatrix} 6 & 6 \\ 2 & 2 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}. \quad \underline{v}_2 = \begin{pmatrix} -1 \\ 1 \end{pmatrix} \qquad \tfrac{1}{2}\text{ pt}$$

Dus

$$\begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = c_1 \underline{v}_1 e^{\lambda_1 t} + c_2 \underline{v}_2 e^{\lambda_2 t}$$

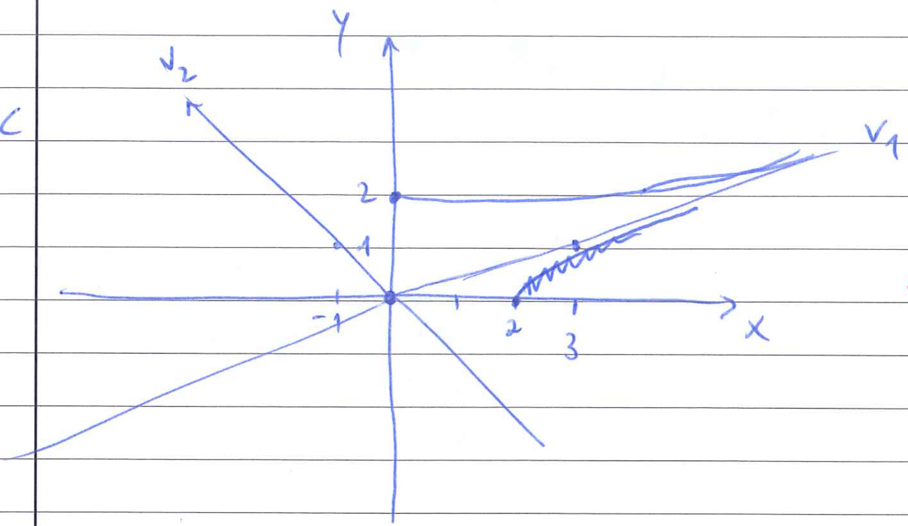
$$= c_1 \begin{pmatrix} 3 \\ 1 \end{pmatrix} e^{4t} + c_2 \begin{pmatrix} -1 \\ 1 \end{pmatrix} e^{-4t}. \qquad \tfrac{1}{2}\text{ pt}$$

b. Op te lossen: c_1 & c_2 zodat

$$\begin{pmatrix} x(0) \\ y(0) \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \end{pmatrix} \iff \begin{pmatrix} 3 \\ 1 \end{pmatrix} c_1 + \begin{pmatrix} -1 \\ 1 \end{pmatrix} c_2 = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$$

$$\iff \begin{pmatrix} 3 & -1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \end{pmatrix} \; \tfrac{1}{2}\text{ pt} \iff c_1 = \frac{1}{2}, \quad c_2 = \frac{3}{2}. \; \tfrac{1}{2}\text{ pt}$$

c



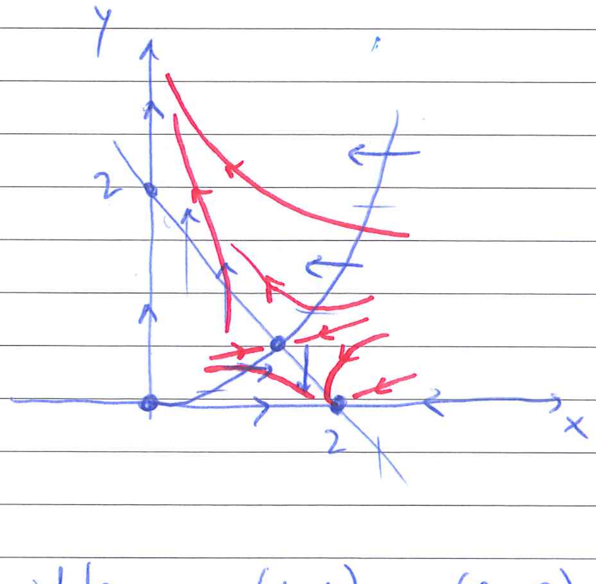
1 pt

d v_1 is de instabiele variëteit
v_2 is de stabiele variëteit

1 pt

2 $\begin{cases} x' = x(2-x-y) \\ y' = y(y-x^2) \end{cases}$, $x \geq 0$, $y \geq 0$

a/c



1 pt (a)

2 pt (c) (rode lijnen)

b. Evenwichten in eerste kwadrant: $(1,1)$, $(2,0)$, $(0,0)$. 1 pt

Jacobiaan: $J = \begin{pmatrix} 2-2x-y & -x \\ -2xy & 2y-x^2 \end{pmatrix}$ $\frac{1}{2}$ pt in $(0,0)$: $J_{(0,0)} = \begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix} \Rightarrow \lambda = 0,\ \lambda = 2$

stabiliteit onbeslist. $(0,0)$ lijkt een niet-geïsoleerd fixpunt.

$\frac{1}{2}$ pt

in $(2,0)$: $J_{(2,0)} = \begin{pmatrix} -2 & -2 \\ 0 & -4 \end{pmatrix} \Rightarrow \lambda = -4,\ \lambda = -2$ $\frac{1}{2}$ pt

stable node.

in $(1,1)$ $J_{(1,1)} = \begin{pmatrix} -1 & -1 \\ -2 & 1 \end{pmatrix}$ $\lambda = \pm\sqrt{3}$ $\frac{1}{2}$ pt

zadelpunt

(dus instabiel

c zie links.

3. $x_{n+1} = r x_n e^{-x_n} = r f(x_n)$.

a. Op te lossen: $rf(x) = x$

$rxe^{-x} = x$ $x = 0$ (het triviale punt)

of $re^{-x} = 1$

$e^{-x} = \frac{1}{r}$

$-x = \log\left(\frac{1}{r}\right) = \log r^{-1} = -\log r$ $1\frac{1}{2}$ pt

Dus $x = \log r$.

$x > 0 \Leftrightarrow r > 1$. $\frac{1}{2}$ pt

b. x^* stabiel $\Leftrightarrow \left| \frac{d}{dx} r x^* e^{-x^*} \Big|_{x = x^*} \right| < 1$

$$f'(x) = e^{-x} - xe^{-x} = e^{-x}(1-x)$$

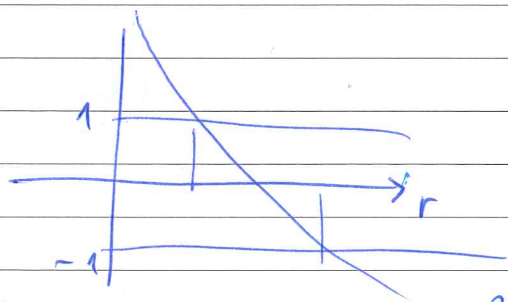
Dus op te lossen:

$$\left| re^{-x}(1-x) \Big|_{x = \log r} \right| < 1.$$

merk op: $e^{-\log r} = e^{\log \frac{1}{r}} = \frac{1}{r}$.

Dus $re^{-\log r} = 1$

$\Rightarrow |1 - \log r| < 1$ 1 pt



linkergrens:

$1 - \log r = 1$

$-\log r = 0$

$r = 1$

rechtergrens:

$1 - \log r = -1$

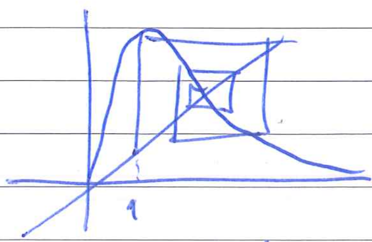
$2 = \log r$

$r = e^2$.

Dus als $r \in (1, e^2)$ dan is $x = \log r$ stabiel

1 pt

c. $r = 5 \Rightarrow 5 \in (1, e^2) \Rightarrow x = \log 5$ is stabiel. 1 pt



(Zolang de iteraties maar overeenkomen met 3b! is het goed)

2 pt

½ pt

d. x^* superstabiel $\Leftrightarrow rf'(x^*) = 0$ ½ pt. Hier: $1 = \log r \Rightarrow r = e$